Project selection and risk taking under credit constraints

Felipe S. Iachan*
FGV/EPGE

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Abstract

We analyze project selection and financing under endogenous credit constraints from limited enforcement. These constraints generate a hedging motive that determines not only borrowing and risk management, but also distorts real investment decisions. Three factors influence this motive: future expected productivity, leverage possibilities, and current net worth. While constrained firms behave as if averse to transitory fluctuations in net worth, they display a risk-loving attitude towards credit capacity and persistent productivity shocks. Thus, they not only fail to insure against these two , but also decide in favor of projects with larger exposure to these underlying risks.

Keywords: capital budgeting, credit constraints, limited commitment, project selection, volatility, exposure, risk.

1 Introduction

Potential difficulties in funding profitable activities are the main justification for corporate hedging, both in theory and practice. In the presence of credit frictions, a hedging motive emerges, prompting firms to better align investment opportunities and available funds even when costly. While the literature has mostly focused on hedging using financial instruments, this hedging motive also affects the way in which firms evaluate the risks inherent to any real investment. When contemplating investment alternatives, firms face projects that might be safer or riskier, correlate more or less with their core business, and offer more stable or unstable financing possibilities. Unlike financial hedging, which affects the distribution of risks across the economy, project selection can actually create aggregate exposure and have larger macroeconomic implications. Yet, little is known about how financial constraints translate into distortions in corporate attitudes towards risk and in capital budgeting.

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Let us illustrate how project selection determines risk exposure with a stylized example in the airline industry. There, a firm might decide to expand into a specific route. By doing that, it is exposing itself to demand fluctuations which determine the prices it can charge and occupancy rates it can achieve for a given airplane. A choice of operating a Boston-New York route exposes revenues from operations to dramatically different risks relative to a choice of operating a route between Phoenix and Las Vegas, even when using the same airplane and crew structure. Other decisions involve the kind of capital used: a more fuel efficient plane should expose the airline less to fluctuations in fuel prices. On the other hand, a plane that is more efficient for a specific route might be less redeployable due to a thinner secondary market, making it less useful as collateral for borrowing against. As such, it could expose the firm to more risk in its ability to secure financing.

Our aim is to formally study the interaction between constrained access to funding, corporate attitudes towards risk, and project selection. Towards that end, we propose a model in which neoclassical firms choose which projects to undertake understanding that for each mix of projects there is an optimal financial plan associated. This plan describes investment levels, borrowing, and hedging policies. Projects are allowed to differ in the revenues they generate, how these react to uncertainty, and also in the type of capital they use. The key financial constraints come from limited enforcement of repayment promises and can be rewritten as simple collateral constraints, as in Rampini and Viswanathan (2010).

Firms which find themselves effectively constrained and unable to reach the efficient investment levels, distort both real and financial investment decisions to reflect the value of internal funds across time and states of the world. When contemplating alternative projects, they also go beyond the evaluation of cash flows from operations and place a premium on a project's ability to attract cheaper collateralized financing.

The main contribution of this paper is in better characterizing the value of funds for constrained firms and in understanding how it affects their attitudes towards risk, ultimately influencing their project selection. This value is shown to consist of a forward looking product of marginal returns, until a moment in which the firm finds itself unconstrained or investment opportunities expire. It is not a pure return on investment that matters, however, but a return on internal funds. On this return, credit capacity and leverage play a central role. Overall, there are three effects that work through the value of internal funds: expected productivity, leverage ability, and current net worth. We illustrate how each one works separately.

We study the effects of both transitory and persistent productivity shocks, which can be understood broadly to encompass the composition of total factor productivity, input costs, and demand fluctuations. Persistent shocks make firms risk loving in the following sense: an increase in exposure to risk through a mean preserving spread increases the value of the firm. The channel which generates this effect relies on self-financing. A persistent productivity improvement makes the firm demand more capital, without generating enough additional cash-flow to fund it. As a consequence, the firm finds itself relatively more constrained after positive news about productivity than after negative news. It is willing to pay a higher premium for additional funds which are available after improvements to productivity. An investment project that loads additionally on this persistent risk is actually more valuable since it generates cash-flows which are better aligned with investment opportunities.

This effect is exactly reversed regarding transitory shocks. These are fully uninformative about future investment opportunities and as such only create a mismatch between the availability of internal funds and target capital levels. While constrained firms might fail to use financial instruments to hedge against those risks, they are willing to distort their real decisions when that reduces the loading on transitory shocks.

Additionally, we provide a condition to be evaluated in order to understand if the behavior of a firm subject to serially autocorrelated productivity shocks is analogous to the one subject to fully persistent shocks rather than the transitory case. Indeed, that condition is easily satisfied in the most common estimates found in the literature[1], indicating that constrained firms face an additional motive for risk taking. This indicates that endogenous project selection has the potential to help account for the higher volatility of constrained firms, which are typically understood to mean lower net worth, younger firms, more prevalent in countries with less developed financial markets.

We also evaluate risk attitudes regarding exposure to the tightening of credit constraints. Constrained firms are concerned about levered returns, which rise when more leverage can be undertaken. They see a higher premium on resources which are available in states with slacker credit conditions, since leverage is higher there. As such, they do not have incentives to ensure resources for situations in which credit conditions deteriorate and do not ensure against a credit cycle. Indeed, projects that show more exposure to credit conditions increase the value of the firm: a project that is more exposed receives more resources exactly when more leverage can be undertaken and loses credit when the opposite is true.

Relation to the literature- In its approach towards financial contracts, the current paper follows Rampini and Viswanathan (2010, 2013) which propose a model in which enforcement constraints can be reduced to collateral constraints, similar to Kiyotaki and Moore (1997), but allowing for the trading of state-contingent assets. The focus of those papers is in analyzing financing and risk management. Capital budgeting, as in most of literature studying financial frictions, is reduced to the choice the of scale of investment[2]. They have had success in explaining some empirical regularities and previously puzzling facts, such as the absence of risk management for the firms typically understood to be the most

[1] The condition is that the sum of the elasticity of revenues to scale and the persistence of log productivity exceeds one. Few papers, actually estimate both jointly. Cooper and Haltiwanger (2006) find an elasticity to scale of 0.89 and a persistent of 0.59. Khan and Thomas (2003) identify an elasticity of 0.9 and persistence of 0.92. In other papers, each value is typically well above 0.5. Midrigan and Xu (2013) find a relatively low persistence of 0.25 in comparison to other studies, but assume constant returns to scale. Collard-Wexler, Asker, and De Loecker (2011) study firms across multiple countries and identify a mean persistence of 0.85, again assuming constant returns to scale.

[2] Consider for instance Albuquerque and Hopenhayn (2004); Bolton, Chen, and Wang (2011); Clementi and Hopenhayn (2006); DeMarzo, Fishman, He, and Wang (2012); He and Krishnamurthy (2012); Holmström and Tirole (1998); or Krishnamurthy (2003).

constrained and the cross sectional profile of leasing and capital purchasing decisions[3]. One of their most emphasized points is that concerns regarding financing investment might be so strong that the most constrained firms fail to hedge altogether.

The contribution of this paper relative to this literature is twofold. As an intermediate step, it decomposes the hedging motive that emerges when financial decisions are constrained. While the risk management literature has particularly emphasized the role of variations in net worth in shaping this motive, it has devoted less attention to state-contingent factors behind the marginal value of funds. We shed light on its dependence on both the expected productivity of a marginal investment, which is intrinsically related to the persistence of shocks, and on leverage possibilities, which are related to external credit conditions.

More importantly, we study selection across alternative investment projects and provide novel results on the capital budgeting decisions of financially constrained firms. We show that a hedging motive is present even for the firms that choose to borrow as much as possible and refrain from financial hedging. It is especially, but not only, the firms that are not seen using the typical financial hedging instruments, such as derivatives, that are the ones expected to display distortions in how they evaluate risks embedded in real investment decisions. These distortions are not always towards safer projects, however.

The current paper is also related to a literature on capital budgeting in environments with frictions. This literature has two main strands. A first one, in the intersection of Finance and Microeconomic Theory, studies distortions in the capital budgeting process of firms that might originate from conflicts of interest between claim holders on the firm and privately informed and self-interested managers. For instance, Harris and Raviv (1996, 1998); Rajan, Servaes, and Zingales (2000); Stein (2002) study difficulties in the allocation of resources to a manager or multiple divisions with conflicting interests[4]. They are concerned about how distortions within the firm might create a problem in allocating resources efficiently. The current paper studies how capital market distortions might feed into distortions in capital budgeting, even in the absence of any internal conflicts.

A second strand comes from a macroeconomic perspective. Stylized examples of project selection have appeared in a literature concerned with aggregate consequences of financial frictions which include Aghion, Angeletos, Banerjee, and Manova (2010), Greenwood and Jovanovic (1990), and Matsuyama (2007, 2008). This paper contributes to that strand in providing a more thorough analysis of the incentives for risk taking and risk management among financially constrained firms, illustrating particular deviations in project selection and novel effects of transitory shocks, persistent ones, and credit fluctuations. The simple assumption of decreasing returns to scale also adds predictions for the behavior along the cross section of firms, not present in the previous literature.

The paper also speaks to a recent literature on endogenous volatility, which has attempted to better

[3]See Rampini and Viswanathan (2013) and Rampini, Sufi, and Viswanathan (2011).

[4]A great survey of work prior to the last decade is available in Stein (2003).

understand how trade-offs faced by firms might help account for the empirical pattern of volatility across countries (Koren and Tenreyro (2013)) and along the business cycle (D'Erasmo and Moscoso Boedo (2013)). This paper adds to that discussion by illustrating first that financially constrained firms would have incentives to load on persistent risks to help self-financing. This not only helps account for some empirical regularities in higher volatility of smaller firms[6], but also points out that increases in risk can actually be an optimal response to the limited access to financing they face. It also offers predictions on how hedging motives would behave for risks of different degrees of persistence.

Some particularly related papers deserve a longer discussion. Vereshchagina and Hopenhayn (2009) study entrepreneurial risk taking in the presence of borrowing constraints. They show that given that entrepreneurs have a real option of stopping their projects and becoming employees in other firms they would become risk loving for sufficiently low wealth. As a consequence, they are willing to choose riskier projects, even in the absence of a premium, which helps account for the surprising low returns found in empirical studies of entrepreneurship when one accounts for the relevant risks and lack of diversification. The result originates from a non-convexity in the value function, which is induced by the occupational choice. As consequence, entrepreneurs are willing to hold more of any risks, even uncorrelated with the productivity of their activity. The risk-taking studied in this paper does not rely on such non-convexity. Value functions are concave state-by-state, but the marginal value of funds is state contingent. Therefore, constrained firms see a hedging motive biasing their decisions and evaluate risks differently depending on how correlated they are with that value.

For the same reason, despite the presence of an enforcement problem, distortions in risk taking do not originate from the same mechanisms as in the risk shifting and asset substitutions literature[5]. Since contracts properly account for possible deviations and assets are observable to lenders, all investment distortions originate from the dispersion in marginal value of funds to the firm and not from a conflict of interest between equity and debt holders.

Another paper, Almeida, Campello, and Weisbach (2011), studies capital budgeting distortions induced by costly access to external funds. It relies on a reduced-form approach describing the choices across a small number of prespecified projects which differ in liquidity and riskiness. It makes a key assumption that projects are uncorrelated. As a consequence, more constrained firms should do more of both financial and operational hedging and end up being less volatile, a result which is at odds with the empirical evidence both across countries with different degrees of financial development and across firm types[6]. The current paper generalizes and qualifies their conclusions by illustrating formally how

[5] In which Jensen and Meckling (1976) is the seminal reference.

[6] Consider the evidence for cross-country comparisons in Koren and Tenreyro (2007) which show that firms in less developed countries concentrate on more volatile sectors and on D'Erasmo and Moscoso Boedo (2013) which draw a comparison of mean volatilities across the larger firm COMPUSTAT versus smaller firms for the Kauffman Firm Survey. Similarly, Davis, Haltiwanger, Jarmin, and Miranda (2007) point out positive correlation between measures of firm growth volatility and size, age, and publicly traded status, usually taken as measures of less severely binding financial constraints.

In the empirical financial development and corporate governance literature, better creditor protection is linked to lower firm level volatility in Claessens, Djankov, and Nenova (2001), which study cross-country firm level evidence, and in Acharya, project changes can be evaluated and how firms react in different ways to shocks which are more or less informative about future opportunities.

Organization- The remainder of the paper is organized as follows. Section 2 reviews the model of financing and risk management which takes a project as given, towards a discussion of the key variable behind firm's decisions, the marginal value of internal funds, in Section 2.1. Project selection is introduced in two ways: through the evaluation of a small-scale marginal project in Section 2.2 and then through the effects induced by a changing project mix in Section 2.3. These general results are then specialized in Section (3), by imposing more structure on specific elements of the model. Examples in Section 3 illustrate how firms evaluate and react to productivity shocks with different degrees of persistence (3.1 and 3.3) and credit capacity shocks (3.2). A final discussion follows.

2 Model

We start by introducing the model of firms' financial decisions, taking their technology or project mix as given. For expositional purposes, the initial set-up revisits Rampini and Viswanathan (2010), a risk management model in which state-contingent borrowing is limited by endogenous collateral constraints. We first use this baseline model to analyze how limited credit, productivity innovations, and leverage possibilities shape the value firms place on funds across states of the world. This marginal value of internal funds is the key variable driving distortions in corporate assessment of risky projects.

Guided by that discussion, we then introduce project selection in Section 2.3. Sections (2.2) and (2.3) provide some general results on how potentially constrained firms evaluate the adoption of a new project and a deviation towards a different mix of projects, respectively. These results are later specialized through examples in the following part of the paper.

The benchmark set up is the following. Time is discrete and indexed by $t = 0, 1, ..., T$, with $T \leq +\infty$. Uncertainty is described by an exogenous event tree. The initial state s^0 is a singleton and $s^t \in S^t$ denotes the history known at time t. We define the transition probabilities between state s^t and its successors s^{t+1}, $\pi\left(s^{t+1}|s^t\right)$, in the usual way and let $\pi\left(s^t\right)$ denote the unconditional probability of state $s^t \in S^t$.

The economy is populated by two types of risk neutral agents. One has access to production technologies and we call them firms. The other group is composed of lenders who, without direct access to a production technology, provide external funding to firms.

A firm maximizes an expected discounted dividend stream according to

$$E\left[\sum_{t=0}^{T} \beta^t d_t\right],$$

Amihud, and Litov (2011), who study creditor protection consequences in M&As, indicating that stronger creditor rights are associated with lower cash-flow risk and more diversification. On the other hand, John, Litov, and Yeung (2008) provide evidence that stronger shareholder rights correlate with reductions in idiosyncratic risk, while controlling for other measures of financial development such as credit/GDP and total market capitalization.

where $\beta \leq 1$. Firms use capital, which is traded at a price $q\left(s^t\right)$. Capital k_{t+1} purchased and installed in state s^t generates $F\left(k_{t+1}, s^{t+1}\right)$ and $(1-\delta)$ units of depreciated s^{t+1} capital. Here $F\left(\cdot, s^{t+1}\right)$ is a standard concave neoclassical production function. In the applications which follow, we look at a separable function $F\left(k_{t+1}, s^{t+1}\right) = A\left(s^{t+1}\right) f\left(k_{t+1}\right)$.

Lenders have a discount factor of $R^{-1} \geq \beta$, are deep-pocketed and not subject to commitment problems, so they are willing to buy and sell contingent claims at an expected rate of return of R. Markets are complete in the sense that assets based on all contingencies can be traded, i.e., a full spanning notion. However, the firm's ability to issue claims on its output is limited by commitment problems.

At date t, after production takes place, a firm can renege on any of its outstanding debt. If that happens, lenders can only recoup a fraction $\theta\left(s^t\right)$ of the firm's capital stock after depreciation for a total value of $\theta\left(s^t\right) q\left(s^t\right)(1-\delta) k_t$[7]. We will refer to $\theta\left(s^t\right)$ as a recovery rate. After reneging on its debt, the firm can go back to capital markets with net worth equal to all of the cash-flows it absconded with plus the fraction $\left(1-\theta\left(s^t\right)\right)$ of the depreciated capital stock.

The problem of a firm with initial net worth w_0 is then of writing a contract choosing a sequence of capital levels, dividends and net payments to lenders $\left\{k_t\left(s^t\right), d_t\left(s^t\right), p_t\left(s^t\right)\right\}_{s^t \in S^t, t \in \{0,1,\ldots,T\}}$ that solves

$$V\left(w_0, s^0\right) \equiv \sup_{\left\{k_t\left(s^t\right), d_t\left(s^t\right), p_t\left(s^t\right)\right\}} E_0\left[\sum_{t=0}^{T} \beta^t d_t\right],$$

subject to resource flow constraints,

$$w\left(s_0\right) \geq d_0 + q\left(s^0\right) k\left(s^0\right) + p\left(s^0\right),$$

$$F\left(k_t, s^t\right) + q\left(s^t\right)(1-\delta) k_t\left(s^{t-1}\right) \geq d\left(s^t\right) + q\left(s^t\right) k_{t+1}\left(s^t\right) + p\left(s^t\right),$$

for each $s^t \in S^t$, $t \in \{1, 2, \ldots, T\}$, the lender's participation constraint,

$$E_0\left[\sum_{t=0}^{T} R^{-t} p_t\right] \geq 0,$$

and enforcement constraints for each $s^t \in S^t$, $t \in \{1, \ldots, T\}$,

$$E\left[\sum_{\tau=t}^{T} \beta^{\tau-t} d_t | s^t\right] \geq V\left(\hat{w}\left(s^t\right), s^t\right),$$

in which $\hat{w}\left(s^t\right) \equiv F\left(k_t, s^t\right) + \left(1-\theta\left(s^t\right)\right) q\left(s^t\right)(1-\delta) k_t\left(s^{t-1}\right)$ is the net worth the firm would abscond with upon default on obligations and $V\left(\hat{w}\left(s^t\right), s^t\right)$ is the supremum of the values the firm could achieve after a deviation at s^t. Notice that the contract allows for any maturity structure of debt.

[7] Allowing for recovery of a fraction of output would not lead to any major departure from the results presented later.

In this environment, if a firm holds $k_{t+1}(s^t)$ units of capital, a natural candidate for a borrowing limit is the maximal amount of resources that could be raised by issuing state contingent debt collateralized by $t+1$ capital, once properly adjusted by the recovery rate and depreciation. A risk neutral lender would be willing to pay $E_t\left[R^{-1}\theta\left(s^{t+1}\right)(1-\delta)\,q\left(s^{t+1}\right)k_{t+1}\left(s^t\right)\right]$ for this set of collateralized claims.

We then define the downpayment required per unit of capital as

$$\varrho\left(s^t\right) \equiv q\left(s^t\right) - E\left[R^{-1}\theta\left(s^{t+1}\right)(1-\delta)\,q\left(s^{t+1}\right)|s^t\right] \tag{1}$$

and financial slack, or unused borrowing capacity as,

$$h\left(s^t\right) \equiv \theta\left(s^t\right)(1-\delta)\,q\left(s^t\right)k_t\left(s^{t-1}\right) - E\left[\sum_{\tau=t}^{T} R^{-(\tau-t)}p_\tau|s^t\right]. \tag{2}$$

In expression (1), the downpayment requirement is defined as the minimum a firm needs to pay in order to deploy a unit of capital, i.e., how much it spends when it finances a purchase at a unit price $q\left(s^t\right)$ by borrowing all that lenders are willing to lend against that collateral. In expression (2), financial slack is the difference between how much the collateral value of the firm's capital is in state s^t, i.e., the borrowing capacity of the firm against that state, and how much the firm is actually pledging to pay from that state onwards. That is, a firm that borrows less than the maximum it could is said to be saving financial slack.

Rampini and Viswanathan (2010) show that the firm's problem can be rewritten in a recursive form[8]. This formulation uses net worth as a state variable and capital, dividend payouts, and financial slack as decision variables. There, the enforcement constraints take the form of collateral constraints, similar to Kiyotaki and Moore (1997) but allowing for state contingency.

The firm's recursive problem can then be written as

$$V_t\left(w_t, s^t\right) = \max_{d_t, k_{t+1}, h_{t+1} \geq 0} d_t + \beta E_t\left[V_{t+1}\left(w_{t+1}, s^{t+1}\right)\right]$$

s.t.

$$w_t \geq d_t + E\left[R^{-1}h_{t+1}\left(s^{t+1}\right)\right] + \varrho\left(s^t\right)k_{t+1}\left(s^t\right)$$

and

$$w_{t+1}\left(s^{t+1}\right) = F\left(k_{t+1}, s^{t+1}\right) + \left(1-\theta\left(s^{t+1}\right)\right)q\left(s^{t+1}\right)(1-\delta)\,k_{t+1}\left(s^t\right) + h_{t+1}\left(s^{t+1}\right).$$

The Envelope Theorem ensures that the multiplier on the first constraint, $\lambda\left(s^t\right)$, equals the shadow value of net worth to the firm, $\frac{\partial V_t}{\partial w_t}$. We will also call it the value of internal funds, interchangeably.

The solution to the recursive maximization problem is then characterized by the following set of

[8] The proof is developed for a case in which the capital recovery is not state contingent, but applies without change to this extension.

first-order conditions:

$$k_{t+1}:\ \beta E_t\left[\frac{\partial V_{t+1}}{\partial w_{t+1}}\left(\frac{\partial F\left(k_{t+1},s^{t+1}\right)}{\partial k_{t+1}}+\left(1-\theta\left(s^{t+1}\right)\right)q\left(s^{t+1}\right)(1-\delta)\right)\right]\leq\varrho\left(s^t\right)\lambda\left(s^t\right), \tag{3}$$

$$d_t:\ 1\leq\lambda\left(s^t\right), \tag{4}$$

and

$$h_{t+1}\left(s^{t+1}\right):\ \beta R\lambda\left(s^{t+1}\right)\leq\lambda\left(s^t\right), \tag{5}$$

each of which holds as an equality if the relevant choice variable is strictly positive.

Equation (3) represents the firm's capital investment Euler equation. Guided by it, we go on to define the levered marginal return on investment as

$$mgR^{lev}\left(k_{t+1},s^{t+1}\right)\equiv\frac{\frac{\partial F\left(k_{t+1},s^{t+1}\right)}{\partial k_{t+1}}+\left(1-\theta\left(s^{t+1}\right)\right)q\left(s^{t+1}\right)(1-\delta)}{\varrho\left(s^t\right)}.$$

This represents the variation in net worth induced by a marginal investment in capital associated to the maximum borrowing possible against that capital as collateral. We rewrite that Euler equation as

$$\frac{\partial V_t\left(w_t,s^t\right)}{\partial w_t}\geq\beta E_t\left[\frac{\partial V_{t+1}\left(w_{t+1},s^{t+1}\right)}{\partial w_{t+1}}mgR^{lev}\left(k_{t+1},s^{t+1}\right)\right]. \tag{6}$$

Notice from equations (4) and (5) that when $\beta=R^{-1}$, a firm that is paying dividends at s^t never becomes constrained again in the future[9]. As a consequence, its investment Euler Equation collapses into

$$R\geq\frac{E_t\left[\frac{\partial F\left(k_{t+1},s^{t+1}\right)}{\partial k_{t+1}}+q\left(s^{t+1}\right)(1-\delta)\right]}{q\left(s^t\right)},$$

with equality when $k_{t+1}\left(s^t\right)>0$. Therefore, for any dividend paying firm, the first-best investment rule is recovered. That is, when investment occurs, its expected marginal returns equals the market interest rate.

The capital accumulation equation indicates the importance of two endogenous variables: the value of internal funds and the marginal levered return. It also indicates that they are intrinsically related. Their behavior is key for understanding how credit constraints influence the decisions of constrained firms, not only in terms of financial planning, but also their real investment decisions. Therefore, before introducing project selection, we have a deeper look at their behavior.

[9] A firm that pays dividends at s^t faces an unitary marginal value of funds at that state, from equation (4). Additionally, $\beta=R^{-1}$ and equation (5) imply that the marginal value of funds is non-increasing. Since equation (4) also implies that ensuring that $\frac{\partial V_{t+1}\left(w_{t+1},s^{t+1}\right)}{\partial w_{t+1}}\geq 1$, it follows that $\frac{\partial V_{t+1}\left(w_{t+1},s^{t+1}\right)}{\partial w_{t+1}}=1$. The same argument can be iterated for any future dates.

2.1 The value of net worth

Standard dynamic programming arguments establish that the value function, $V_t(w_t, s^t)$, is concave in w_t, so that the marginal value of net worth is decreasing[10]. Additionally, when the production function is strictly concave, this marginal value reaches one for sufficiently high net worth.

The concavity of the value function has been pointed out as a reason for risk management, along the lines of the argument first put forward by Froot, Scharfstein, and Stein (1993): financially constrained firms become averse to fluctuations in net worth, since they prevent them from deploying adequate levels of capital across states of the world and create dispersion in the value of internal funds across these states.

A less explored aspect of the marginal value of net worth for the firm lies in its state dependence, as there are other factors besides limited net worth that affect the return on the marginal investment opportunity. To illustrate the mechanics of these factors concisely, it is worth looking at some simple finite time examples.

Let us assume that the firm pays dividends surely at a time $\bar{t} \leq T$. This can occur either as its projects involve a finite life or as an outcome that is reached under the optimal policy for the firm. Then,

$$\lambda_{\bar{t}}\left(s^{\bar{t}}\right) = 1, \, \forall s^{\bar{t}} \in S^{\bar{t}}.$$

Additionally, let us assume that each $F(k_{t+1}, s^{t+1})$ satisfies Inada's conditions. Then, for all $t < \bar{t}$,

$$\lambda_t\left(s^t\right) = \beta^{\bar{t}-t} E_t\left[\prod_{\tau=t+1}^{\bar{t}} mgR^{lev}\left(k_\tau, s^\tau\right)\right]. \tag{7}$$

Therefore, the marginal value of resources within the firm in state s^t depends directly on the composition of the forward levered returns on investment. All else equal, the more constrained, the more levered, and the more productive the firm, the higher these returns are.

To illustrate these effects informally, we resort to a simple three date environment. We let $t \in \{0, 1, 2\}$, the production function be separable as $F(k_{t+1}, s^{t+1}) = A(s^{t+1}) k_{t+1}^{\alpha}$, with $\alpha \in (0, 1)$, and capital be fully pledgeable as in Kiyotaki and Moore (1997), $\theta(s^t) = 1$, for all t, s^t. We focus on $t = 1$, one period before dividends are paid out for sure.

There, whenever $\lambda(s^1) > 1$, the firm is effectively constrained in its capital deployment decisions, and uses maximal leverage, investing all its net worth in capital by purchasing $k_2(s^1) = \frac{w(s^1)}{\varrho(s^1)}$. In that case, the marginal value of internal funds is

$$\lambda_1\left(s^1\right) = \beta\alpha \frac{E\left[A\left(s^2\right)|s^1\right]}{\varrho\left(s^1\right)} k^{\alpha-1}\left(s^1\right) = \beta\alpha \frac{E\left[A\left(s^2\right)|s^1\right]}{\varrho\left(s^1\right)^{\alpha}} w\left(s^1\right)^{\alpha-1}. \tag{8}$$

We can point out three effects in place. The expected productivity term, embedded in $E\left[A\left(s^2\right)|s^1\right]$,

[10] A proof is present in Rampini and Viswanathan (2013).

pushes resources towards being more valuable in higher productivity states. The leverage effect, embedded in the reciprocal of the downpayment requirement, increases the value of resources when the credit conditions are looser and the downpayment is lower. Notice that concavity in the production function works towards dampening this effect, but does not change its sign. Finally, the effect most emphasized in the risk management literature, which originates from the concavity of $f(k_t) \equiv k_{t+1}^{\alpha}$ and makes sure that, ceteris paribus, firms with lower net worth face more severe distortions, deploy less capital, and have higher marginal returns to investment.

We next consider how these three effects interact when firms consider alternative investment projects, as well as how they shape the determination of the optimal financial policies.

2.2 Evaluating a marginal project

Imagine the firm faces an alternative short-term project of small scale. This project requires $\epsilon > 0$ units of a specific capital, which costs $q^{alt}(s^t)$ today. Investment on this project generates a risky cash-flow of $y^{alt}(s^{t+1})\epsilon$, with which the firm could fully abscond in s^{t+1}. It also reverts some $(1-\delta)\epsilon$ units of depreciated capital, which is valued at $q^{alt}(s^{t+1})$ and has a recovery rate of $\theta^{alt}(s^{t+1})$.

As before, we can define two key objects for describing the firm's capital budgeting decisions. The first one is the downpayment requirement for this new project: $\varrho^{alt}(s^t) \equiv q^{alt}(s^t) - R^{-1}E\left[(1-\delta)\theta(s^{t+1})q^{alt}(s^{t+1})\right]$. The second is its marginal levered return of

$$mgR^{alt}\left(s^{t+1}\right) \equiv \frac{y^{alt}(s^{t+1}) + (1-\delta)(1-\theta(s^{t+1}))q^{alt}(s^{t+1})}{\varrho^{alt}(s^t)}. \tag{9}$$

Then, the first-order effects of undertaking that project on the value of the firm, $V_t(w_t, s^t)$, are given by the product of the scale ϵ and

$$\beta E_t\left[\lambda\left(s^{t+1}\right)\left\{y^{alt}\left(s^{t+1}\right) + (1-\delta)\theta\left(s^{t+1}\right)\right\}|s^t\right] - \lambda\left(s^t\right)\varrho^{alt}\left(s^t\right). \tag{10}$$

In a situation in which the firm is involved in production and the Euler Equation for capital investment holds with equality, we get that in the limit in which ϵ tends to zero, the project should be adopted if, and only if,

$$E_t\left[\lambda\left(s^{t+1}\right)\left(mgR^{alt}\left(s^{t+1}\right) - mgR^{lev}\left(s^{t+1}\right)\right)\right] \geq 0. \tag{11}$$

This condition can be rewritten in a covariance form as

$$E_t\left[mgR^{alt}\left(s^{t+1}\right)\right] - E_t\left[mgR^{lev}\left(s^{t+1}\right)\right] + Cov_t\left(\frac{\lambda(s^{t+1})}{E_t[\lambda(s^{t+1})]}, mgR^{alt}\left(s^{t+1}\right) - mgR^{lev}\left(s^{t+1}\right)\right) \geq 0. \tag{12}$$

A few features call attention. First, given that firms cannot borrow arbitrary amounts, project selection is always comparative: at the margin the main project and any alternative compete for internal funds and become mutually exclusive. Firms that are more constrained have higher leveraged marginal returns and, consequently, face naturally higher hurdle rates.

Second, the relevant return that is taken into account is a leveraged return, not a simple return on investment. A project that is capable of raising more collateralized financing requires a lower downpayments and, as a consequence, less resources to be displaced from other profitable opportunities the firm might have.

Third, firms that are constrained take into account a covariance term: projects that pay out more in the states in which the value of internal resources is higher are preferred. A lower return project might be picked over a higher return project if it pays out more in the states in which the firm is more constrained. The example in Section 3.1 illustrates that when productivity is persistent, firms are actually more constrained after positive, rather than negative, productivity innovations. As a consequence, equation (12) would indicate a positive covariance between $\lambda\left(s^{t+1}\right)$ and $mgR^{lev}\left(s^{t+1}\right)$. It follows that diversification away from the baseline project lowers the value of the firm, even if the alternative project offers higher returns.

Notice also that even in the absence of any technological interactions, such as economies of scope, frictions in access to external funding are capable of generating both substitution and complementarity across projects. Substitution is present when two contemporaneous projects which cannot be fully externally financed compete for the use of the firm's resources. Complementarities arise across time, since projects that offer payouts that covary positively with the marginal value of net worth help finance the firm's most productive investment opportunities. Therefore, although the firm is always maximizing the total net present value of dividends, they are not maximizing NPV project-by-project. A project is evaluated in light of its capital requirements, its ability to attract external funding, and its ability of generating additional funding for the most valuable investment opportunities.

Additionally, if we make the discount factor of lenders and firms the same $\beta = R^{-1}$ and look at firms that are effectively unconstrained and paying out dividends at s^t, these face $\lambda\left(s^t\right) = 1$ and $\lambda\left(s^{t+j}\right) = 1$ for any s^{t+j} which is a successor of s^t. Then Equation (12) also collapses back into the first-best rule of optimal investment: a firm should undertake a project if, and only if, it has a $NPV \geq 0$.

2.3 Changes in a firm's project mix

Different investment projects entail different exposures of cash-flows to the most relevant risk factors such as input and output prices as well as productivity shocks, both idiosyncratic and aggregate. They might also differ in other relevant ways such as by involving capital that is more or less redeployable, serves as better collateral, has different exposure to price fluctuations, or different depreciation rates. In our airline example from the introduction, these were embedded in the operational decision among which alternative routes to explore and the aircraft choice.

For tractability, we study how small deviations around a specific project mix affect the value of the firm. Abstractly, we will think in terms of project mixes $j \in \mathcal{J}$, in which $\mathcal{J}$ is a closed interval of the real line. We assume that project selection is a once-for-all decision, which is observable to lenders and can be contracted on.

For concreteness, we allow projects to differ along these three dimensions: how much output is generated in each contingency given capital investment, the price of the capital which is used by the project and its recovery rate[11]. We can think of the first as the exposure of cash-flows to risks, of the second as the fluctuations in the relevant cost of investment/divestment and of the third as sensitivity to variations in credit conditions.

Formally, the consequences of the selection of a different project mix on the value of the firm is a composition of three effects, which we first study separately. We write $F\left(k_t, j, s^t\right)$ for the output function, $q\left(j, s^t\right)$ for the capital price and $\theta_t\left(j, s^t\right)$ for the recovery rate and assume differentiability in j. Therefore, we can think of project selection as either a linear combination of two projects[12], which differ in the type of capital they use and on the stochastic output they generate, in an analogy to a typical portfolio problem, or as the selection of one specific item in a continuum of available alternatives.

The firm's problem involving project selection can be solved in two steps: for each project mix j the optimal financial policy describing borrowing and hedging can be obtained. Let $V_0\left(w_0, j\right)$ denote the value achieved when project j is executed with net worth w_0. Then, the optimal project choice is the solution to $\max_{j \in \mathcal{J}} V_0\left(w_0, j\right)$.

We will proceed by characterizing the effects of small changes of j around a specific mix. We evaluate locally the impacts of changes in the cash-flow process, the prices of capital, and of different recovery rates. The final impact of a different project selection on the value of the firm is a composition of these three effects, which we first illustrate separately. Each of these is obtained by looking at different terms that originate from the use of the Envelope Theorem to study $\frac{\partial V_0(w_0,j)}{\partial j}$.

The part of the effect of a change in the project selection that works through revenue changes is in a composition of envelope effects that add up to

$$\sum_{t=1}^{T} E\left[\beta^t \lambda\left(s^t\right) \frac{\partial F\left(k_t^*, j, s^t\right)}{\partial j}\right].$$

This expression indicates that these cash-flow changes are just evaluated according to the pricing rule implied by the value of net worth to the firm across states of the world.

[11]Input and output price changes can be thought of as comprising part of the fluctuations in the productivity of capital. Changes in depreciation rates and depreciation shocks represent just a small departure from the consequences of capital price changes and will not be discussed.

[12]A generalization for combination of n projects would not require many changes except allowing for $j \in \mathcal{J}$ to be a vector in the $(n-1)$-dimensional simplex.

The effect through capital price changes is a composition of effects that originate from

$$\frac{\partial V_0}{\partial q\left(s^t\right)} = \beta^t \pi\left(s^t\right) \lambda_t\left(s^t\right)\left\{-i^*\left(s^t\right)+\mu\left(s^t\right)(1-\delta)\,\theta\left(j, s^t\right) k_t^*\left(s^{t-1}\right)\right\}, \tag{13}$$

where $i\left(s^t\right) \equiv k_{t+1}\left(s^t\right)-(1-\delta)\, k_t$ is the firm's investment at s^t and

$$\mu\left(s^t\right) \equiv \frac{\lambda\left(s^{t-1}\right)}{\beta R \lambda_t\left(s^t\right)}-1=\frac{E_{t-1}\left[\lambda_t\left(s^t\right) mg R^{lev}\left(k_t^*, s^t\right)\right]}{\lambda_t\left(s^t\right) R}-1 \tag{14}$$

is the premium on borrowing capacity which is perceived by constrained firms. This premium is present for firms that exhaust their debt capacity by pledging as much as possible against capital in state s^t, i.e., for firms that do not leave any financial slack into s^t.

The first term in (13) is the incremental cost of investment, properly adjusted by the value of funds to the firm, while the second term reflects additional borrowing capacity: an anticipated price increase from s^{t-1} to s^t allows for more borrowing at time $t-1$ given the same amount of collateral.

Last, a change in the recovery rate $\theta\left(s^t\right)$ has similar effects,

$$\frac{\partial V_0}{\partial \theta\left(s^t\right)}=\beta^t \pi\left(s^{t+1}\right) \lambda_t\left(s^t\right) \mu\left(s^t\right) q\left(s^t\right)(1-\delta)\, k_t^*\left(s^t\right).$$

That is, it increases the value of any firms that are borrowing constrained, face $\mu\left(s^t\right)>0$ and use all of its borrowing capacity against s^t capital.

In order to combine the three effects in a more meaningful way, we let the borrowing capacity against capital in state s^t be denoted by

$$BC\left(k_t, j, s^t\right) \equiv \theta\left(j, s^t\right)(1-\delta)\, q\left(j, s^t\right) k_t \tag{15}$$

and the firm's net cash-flow be denoted by

$$NCF\left(k_t, i_t, j, s^t\right) \equiv F\left(k_t, j, s^t\right)-q\left(j, s^t\right) i_t. \tag{16}$$

Then, a marginal change in the project mix change is evaluated according to

$$\frac{\partial V_0}{\partial j}=E_0\left[\sum_{t=1}^T \beta^t \lambda\left(s^t\right)\left(\frac{\partial NCF\left(k_t^*, i_t^*, j, s^t\right)}{\partial j}+\mu\left(s^t\right) \frac{\partial BC\left(k_t^*, j, s^t\right)}{\partial j}\right)\right]. \tag{17}$$

Equation (17) indicates that we can think of project selection as involving two terms: an adjusted discounted cash-flow term and a borrowing capacity change. The former is analogous to the usual evaluation of cash-flows from projects, but adjusts properly for the shadow value of internal funds. A novel effect emerges in the latter. A different project mix might change a firm's ability to raise external financing.

Since borrowing is limited by commitment problems, these funds are possibly cheaper than the shadow value of funds for the firm. As a consequence, a premium on borrowing capacity emerges for firms that find themselves against their borrowing constraint. Therefore, projects are not only evaluated according to the net cash-flows from their operation, but also by their ability to attract cheaper collateralized funding.

Going back to the firm's choice among all possible alternatives, we can also use equation (17) to describe the firm's project selection in the following way. Any interior solution needs to satisfy $\frac{\partial V_0}{\partial j} = 0$. This offers is a simple characterization of project selection whenever the optimal choice is interior and (15)-(16) define net-cash flows and borrowing capacity that are concave in j. There we can think in terms of a firm that takes as given the value of internal funds $\lambda\left(s^t\right)$ and the premium on borrowing capacity $\mu\left(s^t\right)$ obtained from the operation of the optimal project and act as if maximizing the sum of discounted cash-flows and premium-adjusted borrowing capacity. Notice that discounting is according to the shadow value of internal funds, not market prices.

Even in more general cases, equation (17) sheds light on which projects cannot be optimal and how different decisions, such as favoring a project with riskier or safer cash flows, can change the value of the firm. We use it for characterizing examples in the next section.

3 Project selection and risk taking

While the previous section included a general description of the environment and the analysis of criteria for evaluating project selection, we now study which qualitative consequences might emerge from distortions which are induced by the hedging motive that is faced by credit constrained firms.

For that purpose, we analyze a few simple examples which illustrate how limited access to external finance changes risk-taking incentives of firms. In all of the examples, we have three dates, $t \in \{0, 1, 2\}$, discount factors that are the same for firms and lenders, $\beta = R^{-1} = 1$, depreciation $\delta \geq 0$, and a separable single-factor neoclassical production function $F\left(k_t, j, s^t\right) = A\left(j, s^t\right) f\left(k_t\right)$, with a smooth and strictly concave $f\left(k_t\right) = \frac{k_t^\alpha}{\alpha}$ for $\alpha \in (0, 1)$.

3.1 Persistent Productivity

For simplicity, let capital be fully collateralizable in all periods and states of the world, so that $\theta\left(j, s^t\right) = 1$ always, and let $\delta > 0$. As a consequence, the downpayment requirement is also constant across time and states of the world and we simply call it ϱ.

Uncertainty is fully revealed at $t = 1$, where it is given by two states only. One state is mapped into higher productivity, while the other is mapped into lower productivity. Since $F\left(k_t, j, s^t\right)$ denotes the revenue generated, we can think of productivity broadly in terms of revenue generation given a capital investment. As such, it incorporates not only changes in output TFP, but also in a reduced-form way any changes in demand conditions, competition, and factor prices. We call the two underlying states $s^1 = s^1_h, s^1_l$. From $t = 1$ into $t = 2$ the event tree evolves trivially: there is a singleton as a successor of either s^1. We refer to them as $s^2 = s^2_h, s^2_l$. We impose an order, saying that $s^t_h > s^t_l$, so states are ordered in terms of the productivity they induce.

We let $A\left(j, s^t_h\right) > A\left(j, s^t_l\right)$ for $t = 1, 2$ and every project j. We first look at the case in which these productivity states are fully persistent, then later revisit this result. In this environment, project selection fully refers to the exposure to this fundamental risk in $s^t = s^t_h, s^t_l$. Our initial step is to characterize the decisions for a fixed project mix, $j = \bar{j}$, for which $A\left(\bar{j}, s^1_i\right) = A\left(\bar{j}, s^2_i\right)$ for $i = h, l$. We later show that projects with higher exposure to productivity risk are more desirable for constrained firms, as they better facilitate their own funding.

Proposition 1. *There exists $\underline{w}$ such that firms facing $w_0 < \underline{w}$ do not save financial slack into either state in $t = 1$ and face values of internal funds which are strictly increasing with respect to s^t. That is, $\lambda_0\left(s^0\right) > \lambda_1\left(s^1_h\right) > \lambda_1\left(s^1_l\right)$.*

Proposition 1 indicates that sufficiently constrained firms value additional funds at $t = 0$ above the value they attribute to funds in either state at $t = 1$. Thus, funding of investment outweighs any risk management concerns and no financial slack is kept for $t = 1$. Notice however that although these firms borrow as much as possible, and don't do any financial hedging, a hedging motive is still present. Firms value strictly more resources at s^1_h than at s^1_l. This is a force towards the distortion of capital budgeting decisions that we explore further. Before proceeding, we impose an additional assumption that allows for a simple characterization of the risk attitudes and hedging behavior of the whole cross section of firms.

Assumption 1. $A\left(s^1_h\right) \alpha^{\frac{1-\alpha}{\alpha}} > E_0\left[A\left(s^1\right)\right]$.

In words, Assumption 1 requires the elasticity of scale to be sufficiently close to one, given the dispersion in $A\left(s^1\right)$[13]. Its main consequence is in ensuring that a firm which deploys the first-best level of capital from $t = 0$ into $t = 1$ can still find itself constrained at $t = 1$ if productivity is revealed to be high. Under this assumption, the behavior of the whole cross-section of firms that differ in initial net worth w_0 can be described in the proposition below, which is illustrated by Figure 1.

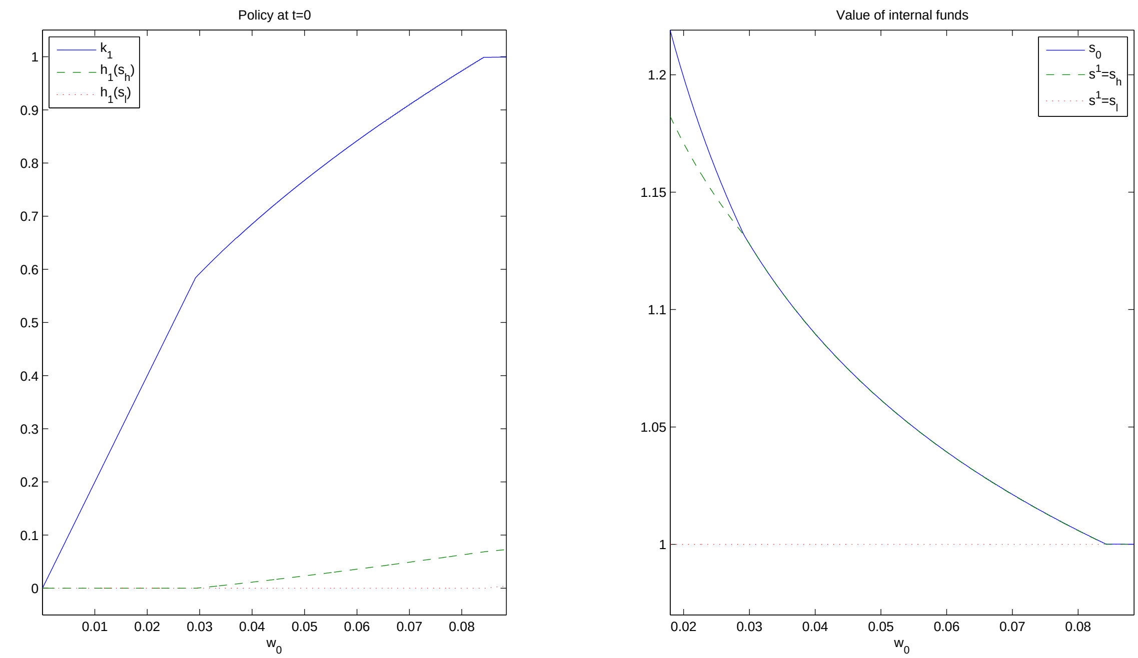
Proposition 2. *Consider the environment just described, with a fixed project mix $j = \bar{j}$ and Assumption 1 satisfied. Then, there exist two thresholds $\underline{w}, \overline{w}$ for initial net worth such that:*

1. *Firms with low net worth, $w_0 \leq \underline{w}$, do not save any financial slack and face shadow values of internal funds which are pro-cyclical with respect to their output, i.e., $\lambda_1\left(s^1_h\right) > \lambda_1\left(s^1_l\right)$.*

2. *Firms with intermediate initial net worth , $\underline{w} < w_0 \leq \overline{w}$, only save financial slack for growth at $t = 1$ when s_h is learned. They also face $\lambda_1\left(s^1_h\right) > \lambda_1\left(s^1_l\right)$.*

[13] For instance, if $A\left(s_h\right) > 1.05E\left[A\left(s\right)\right]$, then any $\alpha > 0.82$ makes sure the assumption is satisfied.

3. Firms with sufficiently high net worth, $w_0 > \overline{w}$, always finance the first-best capital investment level and face $\lambda_0 = \lambda_1\left(s_h^1\right) = \lambda_1\left(s_l^1\right) = 1$.



(a) Capital investment and financial slack.

(b) Value of internal funds.

Figure 1: Capital purchases, financial hedging and value of funds as a function of initial net worth.

Given that characterization, we ask the following question: when project selection refers to trade-offs regarding riskiness in the sense of the dispersion of $A\left(j, s^1\right)$ across states, how do different firms face these trade-offs?

Proposition 3. *When projects differ only on the distribution of t=1 productivity, $A_1\left(j, s_t\right)$, firm's evaluate marginal changes in the project mix according to*

$$\left.\frac{\partial V_0}{\partial j}\right|_{j=\bar{j}} = \frac{\beta}{\alpha} E\left[\lambda_1\left(s_1\right) \frac{\partial A_1\left(\bar{j}, s_1^t\right)}{\partial j}\right] k_1^{\alpha}. \tag{18}$$

Therefore, if j indexes projects by the dispersion of $A_1\left(j, s_t\right)$ around a fixed mean, the effects of an increase in j the firm's value are given by

$$\left.\frac{\partial V_0}{\partial j}\right|_{j=\bar{j}} \propto \left[\lambda\left(s_h^1\right) - \lambda\left(s_l^1\right)\right] k_1^{\alpha} \geq 0. \tag{19}$$

Thus, the value of a firm which is constrained below first-best level capital utilization increases in the dispersion of $t = 1$ productivity, while unconstrained firms are neutral to such increase.

This result illustrates that constrained firms become risk takers with respect to persistent risks. The intuition is the following: exposure to persistent risks generates higher cash-flows exactly when it is more valuable to have them, that is, when the firm has important growth opportunities. This is profitable to constrained firms, even when it is matched by a cost in terms of an equal cash-flow reductions on worse states of nature. From the continuity of the value function, firms would even be willing to sacrifice some net present value according to market prices in order to chose projects with better matched cash-flow generation and growth opportunities.

The same is not true when shocks are purely transitory. Under that situation, higher cash-flows translate into higher net worth, but the desired level of capital does not respond at all. As such, they only create a mismatch between desired capital levels and current net worth[14]. A natural question is: how persistent should shocks be in order for the firm to value more resources after learning about a positive shock than about a negative one? Also, how empirically reasonable are the two polar cases regarding persistence?

In the empirical literature, the most common specification for unit-level revenue TFP is given by

$$\ln A_{t+1} = \mu + \rho \ln A_t + \epsilon_{t+1}, \tag{20}$$

with independent and identically distributed ϵ_t. In that equation, ρ is the coefficient indicating persistence of productivity shocks.

For the example we have been studying, there is no need to model $\epsilon_{t=2}$, since the firm pays dividends for sure at $t = 2$ and $\lambda_2\left(s^2\right) = 1$. As a consequence, residual uncertainty after s^1 has no effect on any of the firm's decisions and, for simplicity, can be ignored. We can then entirely focus on the conditional mean of $A\left(s^2\right)$ and impose the following mean reverting specification

$$E\left[A\left(s^2\right)|s^1\right] = \mu\left[A_1\left(s^1\right)\right]^{\rho} E\left[exp\left(\epsilon_{t+1}\right)\right]. \tag{21}$$

Maintaining the same information structure, we just assume that instead of $A_2\left(s^2\right) = A\left(\bar{j}, s^1\right)$ as before, we now have $A\left(s^2\right) = \mu\left[A_1\left(\bar{j}, s^1\right)\right]^{\rho} E\left[exp\left(\epsilon_{t+1}\right)\right]$. This assumption mimics all the consequences of autocorrelation and mean reversion, while maintaining the simple information structure we have been working with. As such, despite assuming that all uncertainty is revealed at $t = 1$ we can still study the effects of intermediate levels of persistence.

[14]This is further explored in Section 3.3, where persistent and transitory shocks are allowed to coexist.

For this case, whenever $1 < \lambda(s^1) < \lambda(s^0)$,

$$\begin{aligned} \lambda\left(s^1\right) &= \mu\left[A_1\left(s^1\right)\right]^{\rho} E\left[exp\left(\epsilon_{t+1}\right)\right]\left(\frac{w\left(s^1\right)}{\varrho}\right)^{\alpha-1} \\ &= \mu\left[A_1\left(s^1\right)\right]^{\rho} E\left[exp\left(\epsilon_{t+1}\right)\right]\left(\frac{A\left(s^1\right)k_0}{\varrho}\right)^{\alpha-1} \propto \left[A\left(s^1\right)\right]^{\rho+\alpha-1}. \end{aligned} \tag{22}$$

An immediate consequence, is that $\lambda(s^1)$ is increasing in the underlying state whenever $\alpha+\rho \geq 1$. We obtain the proposition below.

Proposition 4. *Consider the environment with some mean-reversion described above. Then, whenever the sum of the coefficient of returns to scale* (α) *and the persistence of log productivity* (ρ) *exceeds one, the value of resources to the firm is non-decreasing in the state* s^1.

Indeed, as discussed in the introduction, most empirical studies find high values for both coefficients[15], indicating that the conclusions regarding additional risk-taking for constrained firms are empirically reasonable. We further extend the discussion of the relative effects of persistent and transitory shocks in Section 3.3. Before that, we have a deeper look at the effect of leverage on the value of funds to the firm.

3.2 Credit Capacity Shocks

The environment described in the previous section illustrated how constrained firms place a higher value on internal funds when productivity is higher rather than lower. As such, they do not insure net worth for the lowest productivity states. Shocks to their credit capacity work much in the same way: they reduce the firms' leverage ability and, as a consequence, the return they can make on internal funds. The example in this section illustrates how firms can sacrifice net worth and investment levels in low credit capacity states in order to invest more when credit conditions are more favorable. Again, they opt not to insure against negative shocks. Indeed, additional exposure to such shocks is shown to increase the value of the firm.

To formalize this reasoning, let there be two states that are learned at $t=1$ which we call $s^1 = s_h^1, s_l^1$ and let us study the situation around a fixed project mix $j=\bar{j}$. From $t=1$ into $t=2$ the event tree evolves trivially: there is a singleton as a successor of either s^1. Similarly to the last section, we refer to them as $s^2 = s_h^2, s_l^2$. The states s_h^1, s_l^1 imply a one to one mapping with $\theta_2(\bar{j}, s^2)$, how much lenders expect to recover if the firm decides to walk away from its debt right after production at $t=2$. We let

$$0 < \theta_2\left(j, s_l^2\right) < \theta_2\left(j, s_h^2\right) < 1.$$

Variation in these recovery rates changes how much credit can be obtained against the same collateral

[15] See, for instance, Collard-Wexler, Asker, and De Loecker (2011); Cooper and Haltiwanger (2006); Khan and Thomas (2003); Midrigan and Xu (2013). A longer discussion is available in footnote 1.

in a way that is orthogonal to any movements that could be happening in collateral prices. It is the simplest way to introduce a credit cycle which is unrelated to the productivity of investment.[16] All other variables in the environment are constant across projects, time and states. Productivity is constant, $A(j, s^t) = A$ for all $t, s^t \in S^t$, and $j \in \mathcal{J}$. For simplicity, we also let $\theta_1(j, s^1) = \theta_1$ for all $s^1 \in S^1$ and $j \in \mathcal{J}$. Depreciation is set to zero, $\delta = 0$.

As a consequence of the variation in the recovery rate, the firm's borrowing capacity given any investment level depends on the underlying state. A firm that invests $k_2(s^1)$ can borrow up to

$$BC(k_t, j, s) = \theta_2\left(j, s^2\right) k_2\left(s^1\right),$$

implying a downpayment requirement that is reduced when credit conditions improve, since

$$\varrho(j, s) = 1 - \theta_2\left(j, s^2\right).$$

The variation in the downpayment requirement is directly responsible for making the return on internal funds increase as credit conditions improve. As a consequence, firms fail both to ensure their investment at s_l^1 and prefer projects that have a higher sensitivity to credit conditions[17]. This is formalized in the next two propositions below.

Proposition 5. *Consider the environment described in the last few paragraphs. Then,*

1. *every firm faces shadow values of net worth which are procyclical with respect to the credit fluctuations, i.e.,* $\lambda_1(s_h) > \lambda_1(s_l)$. *As a consequence, firms never save financial slack towards the low collateralization state.*

2. *capital investment is increasing in* $s_h^1 > s_l^1$, *with* $k_2(s_h^1) > k_2(s_l^1)$.

Proposition 6. *When projects differ only in terms of the dispersion of* $\theta_2(j, s^2)$ *around a same mean, with* $j \in \mathcal{J}$ *indexing this dispersion, then an increase in the dispersion of credit shocks increases the value of the firm, as*

$$\frac{\partial V_0}{\partial j} \propto (\lambda_1(s_h) - 1) k_2(s_h) - (\lambda_1(s_l) - 1) k_2(s_l) > 0. \tag{23}$$

There is a simple interpretation of the effects identified in equation (23) above. When credit constraints are relaxed at s_h^t, the firm can borrow more for every purchased unit of capital. This borrowing generates funds valued at $\lambda_1(s_h^1)$, a value that exceeds the cost of their repayment at $t = 2$, where $\lambda_2(s_h^2) = 1$.

[16] Modeling a credit fluctuation through a change in the recovery rate $\theta\left(s^{t+1}\right)$ instead of $q\left(s^{t+1}\right)$ has the advantage of generating a pure credit fluctuation, since investment in capital produces both output and some capital after depreciation at $t+1$. Therefore, a reduction in $q\left(s^{t+1}\right)$ directly makes investment less productive.

Although it is hard to motivate changes in recovery rate varies along the business cycle, we can interpret shocks to this variable as any shocks that affect how much a lender is willing to offer against a given amount of collateral. For instance, a deterioration of adverse selection in credit markets would have similar effects.

[17] In the sense of having a higher variance of θ_2 for a given mean.

There are two reasons for why the relaxation of borrowing constraints at s_h^2 more than offsets an equivalent tightening at s_l^2. The first one is that the value of being able to borrow more for each unit of capital is higher at s_h^1 than at s_l^1. The second is that the increase in borrowing interacts with more units of capital, since leverage is higher at s_h^1.

3.3 Coexistence of risk management and risk-taking behavior

We now allow the productivity of investment to incorporate both a persistent and a transitory component, which jointly describe the current state. We can think about the transitory shock as pure cash-flow fluctuations, while about the persistent component also representing news about future profitability of investment. A typical example of a pure cash-flow fluctuation is an equipment failure that induces a temporary halt in production, while examples of more persistent shocks are represented by entry by a close competitor, demand fluctuations, or long-lived changes in input prices. We show that firms display a risk-averse attitude relative to the transitory component, while a risk taking attitude relative to the persistent one.

Again, we look at environments in which all uncertainty is settled at $t = 1$, so that the event tree evolves towards singleton successors at $t = 2$. We call the events at $t = 1$, $s^1 \in \{\Delta_{p,h}, \Delta_{p,l}\} \times \{\Delta_{t,h}, \Delta_{t,l}\}$ and use the same notation for their successors at $t = 2$. The first component refers to the persistent element of productivity, while the second refers to a transitory component that fully disappears at $t = 2$. We impose orders on each component by saying that $\Delta_{p,h} > \Delta_{p,l}$ and $\Delta_{t,h} > \Delta_{t,l}$.

In particular, we first look at a fixed project $\bar{j}$ such that

$$A\left(\bar{j}, s^1\right) = \overline{A}\left(1 + \Delta_p\left(s^1\right)\right)\left(1 + \Delta_t\left(s^1\right)\right) \tag{24}$$

and

$$A\left(\bar{j}, s^2\right) = \overline{A}\left(1 + \Delta_p\left(s^2\right)\right), \tag{25}$$

$E\left[\Delta_{p,i}\right] = E\left[\Delta_{t,i}\right] = E\left[\Delta_{p,i}\Delta_{t,i}\right] = 0$, $\Delta_{p,h} > 0 > \Delta_{p,l}$ and $\Delta_{p,h} > 0 > \Delta_{p,l}$. For simplicity, we set $\delta > 0$ and $\theta\left(j, s^t\right) = 1$, constant across projects, time and states of the world, so that capital is fully collateralizable. As a consequence, the downpayment requirement is $\varrho = \delta$, constant.

We can write the shadow value of net worth in state s^1 as

$$\lambda_1\left(s^1\right) = \max\left\{\frac{E\left[A\left(s^2\right)|s^1\right]}{\varrho}\left(\frac{A\left(s^1\right)k_0^\alpha + h_1\left(s^1\right)}{\varrho}\right)^{\alpha-1}, 1\right\}. \tag{26}$$

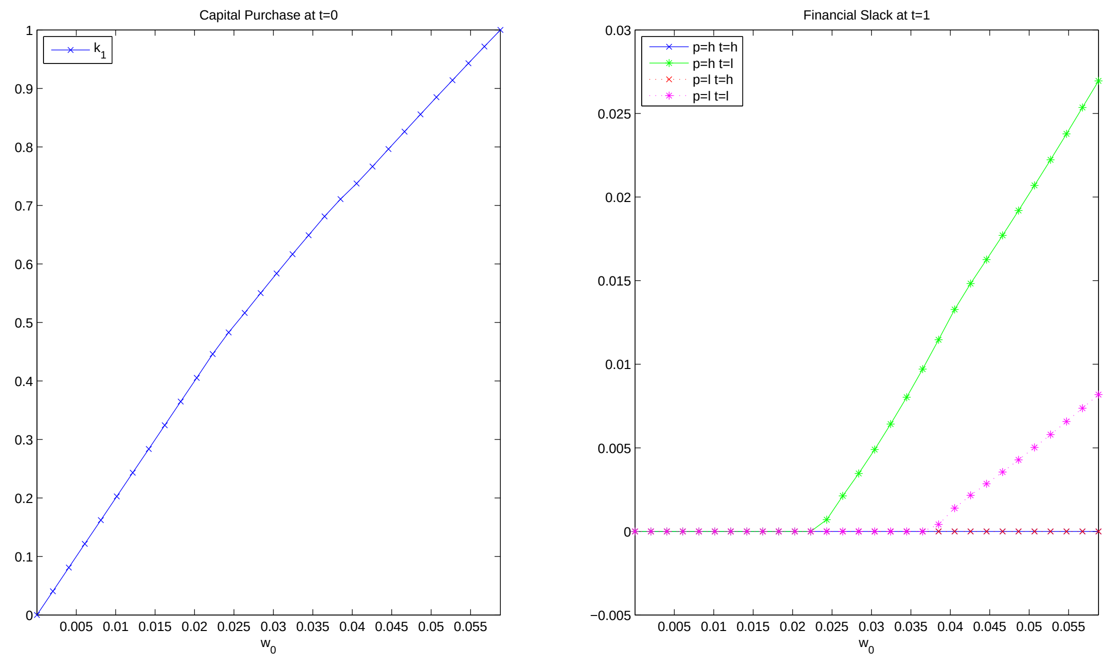
The first entry reflects the marginal levered return when all net worth in state s^1 is invested in capital, using the maximal possible leverage. The second entry reflects the case in which net worth is sufficiently high, capital deployment is unconstrained and the firm is willing to pay out dividends. Looking at the first term, one notices that it encompasses effects through the marginal expected productivity and through the current level of net worth. Increasing $E\left[A\left(s^2\right)|s^1\right]$ should raise $\lambda\left(s^1\right)$, while increasing net worth should lower it.

A persistent shock has the effect of raising both current net worth and the expectation of $A\left(s^2\right)$. As, in Section 3.1, the increase in $A\left(s^2\right)$ dominates any increase in net worth. For a negative transitory shock, however, all the consequences are through decreasing current net worth. A negative transitory shock makes the firm more constrained. These comparisons are formalized in the proposition below.

Proposition 7. *The marginal value of net worth is increasing in the persistent component of productivity shocks and decreasing in the transitory component.*

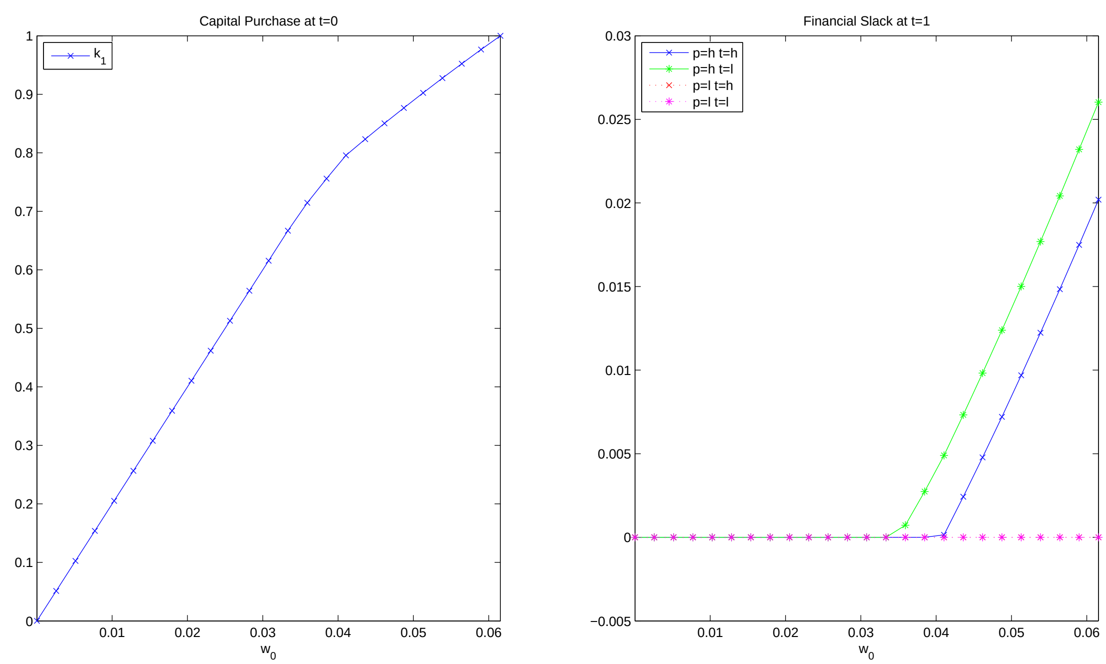
We illustrate the optimal policies for risk management with two examples. In the first example, depicted in Figure (2), the transitory shock is sufficiently larger than the persistent shock. As a consequence, we can see that the hedging of short-term cash-flow fluctuations outweighs the effects of productivity news. All but the most constrained firms insure against a bad cash-flow realization. There is never the saving of financial slack to the state that displays high realizations for both the persistent and the transitory components of productivity, since in that state investment can be fully financed out of the exceptionally high cash flows. Although firms that are sufficiently unconstrained want to insure against negative transitory cash-flow shocks by saving financial slack, they do it to different extents across states, given that market completeness allows them to exploit the sufficiently rich asset structure to better match funds and investment opportunities.

In the second example, illustrated in Figure (3), the growth effect dominates. All but the most constrained firms make sure that financial slack is saved to allow for expansion when good news about a persistent productivity shock emerge. Again, these firms exploit the richness of instruments available and save less for the expansion that is associated with higher cash-flows.



(a) (b)

Figure 2: Optimal investment and financial policies. Large dispersion for transitory component, relative to persistent.



(a) (b)

Figure 3: Optimal investment and financial policies. Equal dispersions for persistent and transitory components.

As in Section 3.1, we are interested in how a project's riskiness is evaluated by firms that differ in their net worth and are, consequently, more or less severely constrained. Towards that end, we assume that all projects differ only in their exposure to $t = 1$ productivity risk. Formally, we write that productivity as

$$A\left(j, s^1\right) = A\left(\bar{j}, s^1\right) + \gamma_p\left(j\right)\Delta_p\left(s^1\right) + \gamma_t\left(j\right)\Delta_t\left(s^1\right). \tag{27}$$

Additionally, let $\gamma_p\left(\cdot\right)$ and $\gamma_t\left(\cdot\right)$ be differentiable, weakly increasing and have roots at $\bar{j}$. Therefore, in this setting, projects are indexed by j and an increase in their index is matched to an increase in their $t = 1$ exposure to both persistent and transitory risks. We show that an increase in the exposure to persistent risks increases the value of the firm, while an increase in the exposure to transitory risks lowers the value of the firm.

Proposition 8. *Consider the setting described in this section,*

1. *Let $\gamma_t\left(j\right) = 0$ for every project $j \in \mathcal{J}$. Then, increases in j are matched to increased exposures to persistent risks only and $\frac{\partial V}{\partial j} = \gamma_p'\left(j\right) Cov\left(\lambda\left(s\right), \Delta_{p,i}\right) \geq 0$.*
2. *Let $\gamma_p\left(j\right) = 0$ for every project $j \in \mathcal{J}$. Then, increases in j are matched to increased exposures to transitory risks only and $\frac{\partial V}{\partial j} = \gamma_t'\left(j\right) Cov\left(\lambda\left(s\right), \Delta_{t,i}\right) \leq 0$.*

The effect of increases in exposure to persistent risks originates from the mechanism explored in Section 3.1 and is related to the efficient matching of cash-flow generation and growth opportunities. The effect of increases in exposure to transitory risks originates purely from the concavity of the value function on net worth: random fluctuations in output induce cash-flow gains and losses which make it harder to smooth out the returns from marginal investment across states of the world.

The effect of an increase in exposure to both risks at the same time is theoretically ambiguous, following the intuition from Figures 2 and 3. Notice that whenever α is sufficiently large, meaning that the returns to scale are close to constant, we get that the persistent effects dominates. This follows from the observation that the transitory effects have no influence on the value of internal funds in the limit in which α approaches unity, since then $\lambda_1\left(s^1\right)$ converges to $\max\left\{\frac{A\left(s^2|s^1\right)}{\varrho}, 1\right\}$. Concavity and the apparent aversion towards transitory shocks disappear together, while a risk-seeking behavior with respect to persistent shocks remains.

4 Conclusion

This paper has illustrated how limited enforcement constrains access to external funds and creates a hedging motive which influences project selection even for firms that do not use financial instruments for hedging. Two key objects which are jointly determined, intrinsically linked, and responsible for the distortions were illustrated: the shadow value of internal funds and the levered return on these funds.

While this hedging motive creates a desire to smooth out transitory cash-flow fluctuations, empirically reasonable levels of persistence in productivity shocks make constrained firms more willing to bear risks that are correlated with their productivity processes. Therefore, since this form of risk-taking helps with self financing, this paper illustrates a channel through which more constrained firms become more volatile. Surprisingly, this is an optimal response to the financial constraints they are subject to in the first place.

We have also emphasized the importance of leverage and illustrated that sufficiently constrained firms are willing to take on more risk that is correlated with credit conditions. Leverage makes internal funds complementary to external funds and can make resources more valuable for the firm when credit conditions are slacker. As a consequence, constrained firms might show a risk-taking attitude regarding their exposure to external credit conditions.

Appendix

Example 1

We prove Propositions 1 and 2 through the combination of three lemmas and a final argument.

We can write the s^1 value function as

$$V_1\left(w_1, s^1\right) = \max_{k_2 \leq \frac{w_1}{\varrho}} A\left(s\right) f\left(k_2\right) - \varrho k_2 + w_1 \tag{28}$$

It clearly inherits the strict local concavity of the production function in the region where $w_1 < \varrho k_2^*\left(s^1\right)$, in which $k_2^*\left(s^1\right)$ is the first-best level of capital investment.

We then look at $t = 0$. Given $\beta = R^{-1} = 1$, there is always a solution that sets $d_0 = 0$. So, we write

$$V_0\left(w_0\right) = \max_{k_1, h_1 \geq 0} E\left[V_1\left(A\left(s^1\right) f\left(k_1\right) + h_1\left(s^1\right), s^1\right)\right] \tag{29}$$

s.t.

$$w_0 = \varrho k_1 + E\left[h_1\left(s^1\right)\right]$$

Notice that again V_0 is strictly concave in w_0 until it reaches the linear part of V_1 for both states at some level we define as $\overline{w}$. For $w_0 \geq \overline{w}$, we have first-best capital levels and linearity of the value function in w_0.

Let k_1^*, $k_2^*\left(s^1\right)$ denote the first-best levels of capital, that is, the ones that set $E_0\left[A\left(s^1\right)\right] k_1^{*\alpha-1} = \varrho$ and $E_1\left[A\left(s^2\right)\right] k_2^*\left(s^1\right)^{\alpha-1} = \varrho$, for $s^1 = s_h^1, s_l^1$.

Lemma 1. *In the environment described in Section 3.1,* $\lambda_1\left(s_h^1\right) \geq \lambda_1\left(s_l^1\right)$.

Proof. Suppose towards a contradiction that $\lambda_1\left(s_l^1\right) > \lambda_1\left(s_h^1\right) \geq 1$. Then, $\lambda_0 \geq \lambda_1\left(s_l^1\right) > \lambda\left(s_h^1\right) \implies$

$h_1\left(s_h^1\right)=0$. Thus, $\lambda_1\left(s_h^1\right) \geq \frac{\beta}{\varrho^\alpha} A\left(s_h^1\right)^\alpha k_0^{\alpha-1} > \frac{\beta}{\varrho^\alpha} A\left(s_l^1\right)^\alpha k_0^{\alpha-1} \geq \lambda_1\left(s_l^1\right)$, reaching the desired contradiction. □

We invoke this lemma in order to prove Proposition 1.

Proof. (Proposition 1) Lemma 1 establishes that $\lambda_1\left(s_h^1\right) \geq \lambda_1\left(s_l^1\right) \geq 1$. Additionally, $\frac{\lambda_0}{\lambda_1\left(s_h^1\right)} = E\left[\frac{\lambda_1\left(s^1\right)}{\lambda_1\left(s_h^1\right)} \frac{A\left(s^1\right)}{\varrho}\right] k_0^{\alpha-1}$. We look at the limit in which $w_0 \to 0$, taking into account that $k_0 \leq \frac{w_0}{\varrho}$. We have $\left[\frac{\lambda_1\left(s^1\right)}{\lambda_1\left(s_h^1\right)} \frac{A\left(s^1\right)}{\varrho}\right] \geq \frac{\frac{A\left(s_l^1\right)}{\varrho}}{\frac{A\left(s_h^1\right)}{\varrho}\left(\frac{A\left(s_h^1\right)k_0^\alpha}{\alpha\varrho}\right)^{\alpha-1}} = Ck_0^{\alpha(1-\alpha)}$ for $C \equiv \frac{\frac{A\left(s_l^1\right)}{\varrho}}{\frac{A\left(s_h^1\right)}{\varrho}\left(\frac{A\left(s_h^1\right)}{\alpha\varrho}\right)^{\alpha-1}} > 0$. As a consequence, $\lim_{w_0\to 0} \frac{\lambda_0}{\lambda_1\left(s_h^1\right)} \geq \lim_{w_0\to 0} Ck_0^{-(1-\alpha)^2} = +\infty$. Therefore, since all conditions for the application of the maximum theorem are satisfied, there exists an interval $I \equiv (0, \underline{w}]$, so that for any $w_0 \in I$, $\lambda_0 > \lambda_1\left(s_h^1\right) \geq \lambda_1\left(s_l^1\right)$, implying that $h_1\left(s^1\right) = 0$ for $s^1 = s_h^1, s_l^1$. As an additional consequence, the inequality $\lambda_1\left(s_h^1\right) \geq \lambda_1\left(s_l^1\right)$ also becomes strict. □

The characterization can be strengthened if we impose a technical condition, ensuring that returns to scale are not too fast decreasing, Assumption 1 in the main text. We rely on this assumption for the remainder of this section.

Lemma 2. *$k_2^*\left(s_h^1\right) > \frac{A\left(s_h^1\right)}{\alpha\varrho} k_1^{*\alpha}$ and $k_2^*\left(s_l^1\right) < \frac{A\left(s_l^1\right)}{\alpha\varrho} k_1^{*\alpha}$.*

Proof. Notice that $k_2^*\left(s_h^1\right) = \left[\frac{A\left(s_h^1\right)}{\varrho}\right]^{\frac{1}{1-\alpha}} > \frac{A\left(s_h^1\right)}{\alpha\varrho}\left[\frac{E_0\left[A\left(s^1\right)\right]}{\varrho}\right]^{\frac{\alpha}{1-\alpha}} = \frac{A\left(s_h^1\right)}{\alpha\varrho} k_1^{*\alpha}$, where we use Assumption 1. Analogously, $k_2^*\left(s_l^1\right) = \left[\frac{A\left(s_l^1\right)}{\varrho}\right]^{\frac{1}{1-\alpha}} < \frac{A\left(s_l^1\right)}{\varrho}\left[\frac{E_0\left[A\left(s^1\right)\right]}{\varrho}\right]^{\frac{\alpha}{1-\alpha}} = \frac{A\left(s_l^1\right)}{\varrho} k_1^{*\alpha} \leq \frac{A\left(s_l^1\right)}{\alpha\varrho} k_1^{*\alpha}$. □

Following Lemma 2, we can also define $\underline{h_1}\left(s_h^1\right)$ so that $\frac{h_1^*\left(s_h^1\right)}{\varrho} = k_2^*\left(s_h^1\right) - \frac{A\left(s_h^1\right)}{\alpha\varrho} k_1^{*\alpha} = \left[\frac{A\left(s_h^1\right)}{\varrho}\right]^{\frac{1}{1-\alpha}} - \frac{A\left(s_h^1\right)}{\alpha\varrho}\left[\frac{E_0\left[A\left(s^1\right)\right]}{\varrho}\right]^{\frac{\alpha}{1-\alpha}} > 0$. That is the minimal financial slack that need to be left into s_h^1 so that the first-best capital level can be deployed. Notice that the first-best capital level can always be deployed at s_l^1 if $k_1 = k_1^*$. Therefore, we can write $\overline{w} = \varrho k_0^* + \pi\left(s_h^1\right) \underline{h_1}\left(s_h\right)$. Any firm with $w_0 \geq \overline{w}$ can invest the first-best capital level in all periods and states, while any firm with $w_0 < \overline{w}$ is sure to be unable to invest both k_1^* and $k_2^*\left(s_h^1\right)$.

Lemma 3. *If $\lambda_1\left(s_h^1\right) = \lambda_1\left(s_l^1\right)$, then $\lambda_0 = \lambda_1\left(s_h^1\right) = \lambda\left(s_l^1\right) = 1$.*

Proof. The capital Euler equation implies, $\lambda_0 = E\left[\lambda_1\left(s^1\right) mgR^{lev}\left(k_1, s\right)\right] = \lambda_1\left(s_h^1\right) \frac{E[A(s)]}{\varrho} k_1^{\alpha-1}$.

Case 1: $\lambda_0 = \lambda_1\left(s_h^1\right)$. Then, $1 = \frac{E\left[A\left(s^1\right)\right]}{\varrho} k_1^{\alpha-1} \implies k_1 = k_1^*$. This implies from Lemma 2 that $\lambda_1\left(s_l^1\right) = 1$. Then, $\lambda_0 = \lambda_1\left(s_h^1\right) = \lambda\left(s_l^1\right) = 1$.

Case 2: $\lambda_0 > \lambda_1\left(s_h^1\right)$. Then, $h_1\left(s^1\right) = 0$ for $s^1 = s_h^1, s_l^1$. Also, $\frac{E\left[A\left(s^1\right)\right]}{\varrho} k_1^{\alpha-1} = \frac{\lambda_0}{\lambda_1\left(s_h^1\right)} > 1 \implies k_1 < k_1^*$. Then, $\lambda_1\left(s_h^1\right) \geq \frac{A\left(s_h^1\right)}{\varrho}\left(\frac{A\left(s_h^1\right)k_1^\alpha}{\varrho}\right)^{\alpha-1} > \frac{A\left(s_h^1\right)}{\varrho}\left(\frac{A\left(s_h^1\right)k_1^{*\alpha}}{\varrho}\right)^{\alpha-1} > \frac{A\left(s_h^1\right)}{\varrho}\left(k_2^*\left(s_h^1\right)\right)^{\alpha-1} = 1$, again using

Lemma 2. Then, $\lambda_1(s_l^1) = \lambda_1(s_h^1) > 1$, which ensures $k_2(s_l^1) = \frac{w(s_l^1)}{\varrho}$ in any solution. It follows that

$$\frac{\lambda_1(s_h^1)}{\lambda_1(s_l^1)} = \frac{A(s_h^1)\left(A(s_h^1)k_0^\alpha\right)^{\alpha-1}}{A(s_l^1)\left(A(s_l^1)k_0^\alpha\right)^{\alpha-1}} = \left[\frac{A(s_h^1)}{A(s_l^1)}\right]^\alpha > 1,$$

reaching a contradiction.

□

At $w_0 = \overline{w}$, the solution to the firm's recursive problem at $t = 0$ is unique, featuring $k_1 = k_1^*$, $k_2(s^1) = k_2^*(s^1)$, for $s^1 = s_h^1, s_l^1$, $h_1(s_h^1) = h_1^*(s_h^1)$ and $h_1(s_l^1) = 0$. From Proposition 1, for sufficiently small w_0, we obtain $h_1(s_h^1) = h_1(s_l^1) = 0$. Berge's maximum theorem combined with the strict concavity the objective in (29) in the region $w_0 \leq \overline{w}$ also ensures that $h_1(s^1)$ and k_1 are a continuous functions in this region. As a consequence, there exists $\underline{w} \in (0, \overline{w})$ such that $\underline{w} = \left\{\sup w | h_1(s_h^1) = 0\right\}$. All the statements in Proposition 2 follow from this characterization.

Proof of Proposition 3

The first-order impact of a change in $A(j, s^1)$ on V_0 is described by

$$\frac{\partial V_0}{\partial j} = \frac{\beta}{\alpha} E\left[\lambda\left(s^1\right) k_1\left(s^0\right)^\alpha \frac{\partial A(j, s^1)}{\partial j}\right] = \frac{\beta}{\alpha} E\left[\lambda\left(s^1\right) \frac{\partial A(j, s^1)}{\partial j}\right] k_1\left(s^0\right)^\alpha$$

When projects differ only in terms of dispersion of productivity at $t = 1$, they can be described with $A_1(j, s^1) = A(\overline{j}, s^1) + \gamma(j)\Delta(s^1)$, where $\Delta(s^1)$ is a random variable with zero mean, satisfying $\Delta(s^1 = s_h) > 0 > \Delta(s^1 = s_l)$, and $\gamma(j)$ is some underlying sensitivity function describing how fast the dispersion of productivity increases as j is increased along $\mathcal{J}$. That function has a root at $\overline{j}$, from the definition of $\overline{j}$. Therefore,

$$\begin{aligned}\frac{\partial V_0}{\partial j} &= \frac{\beta}{\alpha}\gamma'(j) k_1\left(s^0\right)^\alpha E\left[\lambda\left(s^1\right)\Delta\left(s^1\right)\right] \propto \frac{\beta}{\alpha}\gamma'(j) k_1\left(s^0\right)^\alpha \left[\lambda\left(s_h^1\right) - \lambda\left(s_l^1\right)\right] \\ &\propto k_1\left(s^0\right)^\alpha \left[\lambda\left(s_h^1\right) - \lambda\left(s_l^1\right)\right]\end{aligned}$$

Example 2

We have $\lambda_1(s^1) = \min\left\{\frac{A\left(\frac{w_1(s^1)}{\varrho_1(s^1)}\right)^{\alpha-1} + \left(1-\theta_2(s^2)\right)}{\varrho_1(s^1)}, 1\right\}$ for $s^1 = s_h^1, s_l^1$ and s^2 being its unique successor. Given that $\varrho_1(s^1) = 1 - \theta_2(s^2)$, it simplifies to $\lambda_1(s^1) = 1 + A\left[w_1(s^1)\right]^{\alpha-1}\left[\varrho_1(s^1)\right]^{-\alpha} > 1$. This expression is decreasing in both $\varrho_1(s^1)$ and $w_1(s^1)$. As a consequence, it is increasing in $\theta_2(s^2)$.

Lemma 4. *In the environment described in example 2,* $\lambda_1(s_h^1) > \lambda_1(s_l^1) > 1$.

Proof. There are two cases to consider: $h_1(s_h^1) = 0$ and $h_1(s_h^1) > 0$. In the former, $w_1(s_h^1) = \frac{A}{\alpha}k_0^\alpha + (1-\theta_0)k_0 \le w_1(s_l^1)$. Then, $\lambda_1(s_h^1) = 1 + A[w_1(s_h^1)]^{\alpha-1}[\varrho_1(s_h^1)]^{-\alpha} \ge 1 + A[w_1(s_l^1)]^{\alpha-1}[\varrho_1(s_h^1)]^{-\alpha} > 1 + A[w_1(s_l^1)]^{\alpha-1}[\varrho_1(s_l^1)]^{-\alpha} = \lambda_1(s_l^1)$. In the latter, we need $\lambda_1(s_h^1) = \lambda_0 \ge \lambda_1(s_l^1)$. If equality between all three multipliers were to happen, the investment Euler equation would establish that $\lambda_0 = \left(1 + A[w_0]^{\alpha-1}[\varrho_0]^{-\alpha}\right)E[\lambda_1(s^1)] \implies 1 = 1 + A[w_0]^{\alpha-1}[\varrho_0]^{-\alpha} > 1$ reaching a contradiction. □

Both statements in Proposition (5) follow from the lemma above. The first one is immediate. For the second one, we argue that since $\lambda(s^1) > 0$ for $s^1 = s_h^1, s_l^1$, we get that firms resort to maximal leverage at $t = 1$ and $k_1(s_h^1) \ge \frac{\frac{A}{\alpha}k_0^\alpha + (1-\theta_0)k_0}{\varrho_1(s_h^1)} > \frac{\frac{A}{\alpha}k_0^\alpha + (1-\theta_0)k_0}{\varrho_1(s_l^1)} = k_2(s_l^1)$ where the last equality follows from the fact that $\lambda_0 \ge \lambda_1(s_h^1) > \lambda_1(s_l^1)$, which ensures that $h_1(s_l^1) = 0$.

Proof of Proposition 6

When changes occur with respect to $\theta(j, s^2)$ only, projects are evaluated according to

$$\frac{\partial V_0}{\partial j} = \beta E\left[\left(\lambda\left(s^1\right) - \frac{\beta\lambda(s^2)}{R}\right)\frac{\partial\theta(j, s^2)}{\partial j}k_2\left(s^1\right)\right],$$

which simplifies further, given that $\beta = R = \lambda(s^2) = 1$. Additionally, we describe projects ordered as mean preserving spreads $\theta(j, s^2)$ with $\theta(j, s^2) = \theta(\bar{j}, s^2) + \gamma(j)\Delta(s^2)$ where $E[\Delta] = 0$ and $\Delta(s^2 = s_h) > 0 > \Delta(s^2 = s_h)$ and $\gamma(j)$ as a smooth increasing function with a root in $\bar{j}$. Then,

$$\begin{aligned}\frac{\partial V_0}{\partial j}|_{\bar{j}} &= \gamma'(j)E\left[\left(\lambda\left(s^1\right) - 1\right)k_2\left(s^1\right)\Delta\left(s^2\right)\right] \\ &\propto \left(\lambda\left(s_h^1\right) - 1\right)k_2\left(s_h^1\right) - \left(\lambda\left(s_l^1\right) - 1\right)k_2\left(s_l^1\right).\end{aligned}$$

Given both statements in Proposition (5), we can sign this as a positive term.

Example 3

Proof of Proposition 7

Notice that

$$\lambda\left(s^1\right) = \begin{cases}\lambda\left(s^0\right) & \text{, whenever } \lambda\left(s^0\right) < \frac{\overline{A}^\alpha(1+\Delta_{p,i})^\alpha}{(1+\Delta_{t,i})^{1-\alpha}}k_0^{\alpha-1}, \\ \frac{\overline{A}^\alpha(1+\Delta_{p,i})^\alpha}{(1+\Delta_{t,i})^{1-\alpha}}k_0^{\alpha-1} & \text{, whenever } \lambda\left(s^0\right) \le \frac{\overline{A}^\alpha(1+\Delta_{p,i})^\alpha}{(1+\Delta_{t,i})^{1-\alpha}}k_0^{\alpha-1} \le 1, \\ 1 & \text{, whenever } \frac{\overline{A}^\alpha(1+\Delta_{p,i})^\alpha}{(1+\Delta_{t,i})^{1-\alpha}}k_0^{\alpha-1} < 1,\end{cases}$$

which is decreasing in $\Delta_{t,i}$ and increasing in $\Delta_{p,i}$.

Proof of Proposition 8

The effect of a change in projects is given by

$$\begin{aligned} \frac{\partial V_0}{\partial j} &= E\left[\lambda\left(s^1\right)\frac{\partial A\left(\bar{j},s^1\right)}{\partial j}\right]\frac{k_1\left(s^0\right)^\alpha}{\alpha} \\ &= E\left[\lambda\left(s^1\right)\left(\gamma_p'\left(\bar{j}\right)\Delta_p\left(s^1\right)+\gamma_t'\left(\bar{j}\right)\Delta_t\left(s^1\right)\right)\right]\frac{k_1\left(s^0\right)^\alpha}{\alpha} \end{aligned}$$

Whenever $\gamma_t\left(j\right)=0$ for all j, riskier projects always include more loading on the persistent factor. Therefore,

$$\frac{\partial V_0}{\partial j}=E\left[\lambda\left(s^1\right)\Delta_p\left(s^1\right)\right]\gamma_p'\left(\bar{j}\right)\frac{k_1\left(s^0\right)^\alpha}{\alpha}=Cov\left(\lambda\left(s^1\right),\Delta_p\left(s^1\right)\right)\gamma_p'\left(\bar{j}\right)\frac{k_1\left(s^0\right)^\alpha}{\alpha}$$

Since $\lambda\left(s^1\right)$ is always higher for states with a component of $\Delta_{p,h}$ instead of $\Delta_{p,l}$, that expression is non-negative.

Analogously, whenever $\gamma_p\left(j\right)=0$, riskier projects load only on the transitory factor and as a consequence

$$\frac{\partial V_0}{\partial j}=Cov\left(\lambda\left(s^1\right),\Delta_t\left(s^1\right)\right)\gamma_t'\left(\bar{j}\right)\frac{k_1\left(s^0\right)^\alpha}{\alpha}\leq 0.$$

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