Money, limited participation and heterogeneity : An alternative view of monetary policy

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VERY PRELIMINARY

Abstract

The paper presents a general equilibrium model where agents have limited participation in financial markets and can use money to smooth consumption. This framework is consistent with recent empirical finding on money demand. Moreover, new developments in the heterogeneous agents literature are used to develop a tractable framework. It is shown that the market allocation is not efficient. Agents can either over or under invest, depending of the persistence of the technology shocks. The optimal monetary policy is characterized. it helps to restore the right incentive to save for participating agents.

JEL : E41, E52, E32

Keywords : Limited participation, money demand, optimal policy.

1 Introduction

Central banks have recently injected an important quantity of money injection in the economy to buy a wide variety of assets. To understand the effect of money injection, one must use realistic models of money demand. The recent empirical literature on money shows that limited participation of households in financial markets is necessary to account for the empirical distribution of money. One of the simplest limited participation model is the Baumol-Tobin model where agents participate infrequently to financial markets. A recent literature, reviewed below, shows that this model, augmented with a richer stochastic structure, accounts well for money demand (with various definitions of money). Whereas a vast literature introduces constraints

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on the goods market to generate a money demand for transaction purpose, this recent literature suggests that the main friction to understand money demand relies on the financial markets, which is the difficulty to exchange rapidly and at a low cost money against interest-bearing financial assets.

These results on money demand are consistent with the literature on incomplete participation model of money. This literature has been first developed to account for the liquidity effect of money injection: an increase in the money supply decreases the nominal interest rate and increases economic activity in the short-run, what is difficult to obtain in a cash-in-advance environment with flexible prices. This literature listed below has to deal with the agents heterogeneity resulting from limited participation to financial markets. For instance, Lucas (1990) uses the assumption of a representative family, which splits within the period. Alvarez, Atkeson and Edmond (2009) assume periodic participation in financial markets.

In this paper, we use both use both the recent empirical literature on money demand and recent developments in heterogeneous agents models to derive new results on monetary policy with incomplete participation to financial markets. For instance, optimal monetary policy has not been analyzed in this framework. This paper presents a model where households do not always participate to financial markets. When they do not participate, households can smooth consumption by holding money. To identify all the effects, a simple model is first analyzed, where a fraction of the population only uses money to smooth consumption, whereas the other fraction uses financial assets. This framework generates some new results.

We first show that this model generates realistic effect of money injection, as identified by Christiano, Eichenbaum and Evans (1996), after a money injection, the nominal interest rate decreases, inflation increases, economic activity, profits and the real wage increases.

Second, the main contribution of this paper is to identify the distortion generated by limited participation. It is shown that the investment decision after a technology shock is not optimal when participation to financial markets is limited. The economy can generate either over-investment or under investment according to the persistence of the technology shock. Indeed, two effects interact after a technology shock. First, agents who only use money to smooth consumption have a low return on their savings, and they thus have the wrong incentive to save in money. This first effect contributed to under-investment. Second, participating agents experience an increase of the per capita wealth after a technology shock. Under realistic intertemporal elasticity of substitution, they tend to save more than the central planner would have done to smooth consumption. This second effect tend to generate over-investment. It is shown that when the persistence of the technology shock is low the first effect dominates (under-investment), and when the persistence of the technology shock is high the second effect dominates (over-investment).

Third, it shown that an active monetary policy is can be welfare improving when participation to financial markets is limited. The first best monetary policy restores the optimal incentives to save and allow thus an optimal capital accumulation. This identification of the optimal monetary policy can be seen as the main contribution of the paper.

Finally, this framework provides also other results of independent interest. It is shown that price can be determinate under an interest rate rule when the central bank does not target inflation but the quantity of money.

2 Related Literature

Money demand. A recent literature has derived new results on money demand using the cross-sectional information about money holdings. The work of Alvarez and Lippi (2009) show that models with limited participation to financial markets can reproduce the distribution of money if one enriches the model with a stochastic access to cash and some random participation to the goods market. Ragot (2013) shows that the distribution of money, even defined in a narrow sense, is much more similar to the distribution of financial asset than to the distribution of consumption expenditures. In a quantitative model where households face both a cash-in-advance constraints and participation cost to financial markets, he shows that limited participation accounts for more than 80% of money demand. Recently Alvarez and Lippi (2013) show that in addition to limited participation, lumpy expenditures is an important feature to reproduce a realistic money demand. Agents face some indivisibilities in their expenditures on the goods market, for which they have to hold sizable amount of money. Building on this results, the model of this paper introduced both limited participation and lumpy expenditures.

Limited Participation. Limited participation models have been introduced to rationalize the liquidity effect of money injections. (Grossman and Weiss (1983) and Rotemberg (1984)). This literature has to deal with households heterogeneity. , Lucas (1990) and Fuerst (1992) use a family structure. Agents within the family are separated at the beginning of the period and join at the end of the period to pool risk. This outcome does not allow for persistent effects of money shocks, which are shown to be crucial in this paper. Some other tools have been introduced. Alvarez, Atkeson, Edmond (2009) use an overlapping-generation structure. The work of Alvarez and Lippi focus now on partial equilibrium to derive some new results on participation rules (and thus equilibrium environment) when households face a rich stochastic structure. Compared to these papers, my simplifies the stochastic structure to allow for general equilibrium analysis.

3 The Simple model

The model features a closed economy populated by a continuum of households indexed by i and uniformly distributed along the unit interval, as well as a representative firm. Every household i is endowed with one unit of labour, which is supplied inelastically to the representative firm if the household is employed. When they consume households have a period utility function $u(.)$ and have a discount factor β. In the simple model, it is assumed that a fraction households Ω do not participate to financial markets and only use money to smooth consumption. The remaining fraction $1-\Omega$ of households participate to financial markets.

3.1 Non-participating households

Non participating households are denoted by the upperscript n. They consume every other period. It is assumed that a fraction $\Omega/2$ consume in odd period and a fraction $\Omega/2$ consume in even period. These households work every period and earn a nominal wage W_t and pay nominal taxes $P_t\tau_t$, where P_t is the price of one unit of final goods and τ_t is taxes in real terms. When households do not consume, their money demand is their total income $M_t = W_t - P_t\tau_t$.When households consume, they spend the sum of their nominal and of all their money holdings:

$$P_t c_t^n = W_t + M_{t-1}^n - P_t\tau_t$$

Upper case letters denote the nominal letter, lower case letters denote real variables. We denote as $\pi_t = P_t/P_{t-1} - 1$ as the net inflation rate. The choice of non-participating agents is

$$m_t^n = w_t - \tau_t/2$$
$$c_t^n = w_t + \frac{m_{t-1}}{1+\pi_t} - \tau_t$$

One can check that the condition for households not to hold money when they consume is

$$u'\left(c_t^H\right) > \beta^2 \frac{1}{1+\pi_{t+1}} \frac{1}{1+\pi_{t+2}} u'\left(c_{t+2}^H\right)$$

The previous condition is always, when the economy is not at the Friedman Rule what will be the case in the equilibrium under consideration.

3.2 Participating Households

We denote participating agents by the upperscript p. The fraction $1-\Omega$ of households who participate in financial markets, where they buy interest bearing assets. It is assumed that these households consume every period. It is direct to introduce period consumption of participating households for them to have the same utility function as non-participating agents, at the cost of more algebra. As a more general model is presented below, we focus here on the simplest case for participating agents. It is assumed that participating households own the firms in the economy. As a consequence, they receive all residual profits of the firms denoted. The total residual profits are denoted χ_t.

The nominal budget constraint of participating agents is

$$B_t^p + P_t c_t^p = R_t (1+\pi_t) B_{t-1}^p + \frac{P_t \chi_t}{1-\Omega} + W_t - P_t \tau_t$$

where B_t^p is the per capital nominal amount of interest-rate bearing asset, c_t^p is real consumption, R_{t-1} is the real interest rate between period $t-1$ and t. As a benchmark case, we assume that markets are complete for the inflation risk and that contracts between period $t-1$ and period t are written in real term in period $t-1$. The case of contracts written in nominal terms in which inflation surprises affect ex post real interest rate is taken as a simple extension. $\frac{P_t \chi_t}{1-\Omega}$ is the per capita net nominal profits received by participating households. Utility maximization yields the Euler equation, and the budget constraint in real terms.

$$u'(c^p) = R_t E_t u'\left(c_{t+1}^p\right) \tag{1}$$
$$b_t^p + c_t^p = R_{t-1} b_{t-1}^p + w_t + \frac{\chi_t}{1-\Omega} - \tau_t$$

3.3 Firms

There is a unit mass of representative firms, which produce with capital and labor. Capital must be installed one period before production and is assumed the depreciation rate is λ. The production function is Cobb-Douglas

$$Y_t = A_t k_{t-1}^{\alpha} L_t^{1-\alpha}$$

where k_t, L_t and A_t are respectively the capital stock the labor hired and the technology shock known at the beginning of period t. Profit maximization yields the real wage, w_t, the real interest rate R_{t-1} and ex post profits χ_t.

$$\begin{aligned} w_t &= (1-\alpha) A_t k_{t-1}^{\alpha} L_t^{\alpha} \\ R_{t-1} &= \alpha E_{t-1} A_t k_{t-1}^{\alpha-1} L_t^{1-\alpha} + (1-\lambda) \\ \chi_t &= \alpha k_{t-1}^{\alpha} \varepsilon_t \end{aligned} \tag{2}$$

It is assumed that $A_t = e^{a_t}$, where a_t follows a AR(1) process

$$a_t = \rho a_{t-1} + \varepsilon_t^a$$

and ε_t^a is $\mathcal{N}(0, \sigma^a)$.

3.4 Open market and budget of the State

The State raises taxes, τ_t, issue some nominal debt B_t and receive the profits of the central banks. The real value of this debt is denoted as $b_t = B_t/P_t$. The real value of the debt is denoted as Θ_t. This revenue is used to finance a public good g_t and to repay its debt $R_{t-1}b_{t-1}$

$$b_{t-1}R_{t-1} + g_t = b_t + \tau_t + \Theta_t$$

For the public debt to be stationary, it is assumed that the State follows a simple fiscal rule and raises taxes when public debt increases. The time-varying tax rate is

$$\tau_t = \varphi R_{t-1}\left(b_{t-1}^p - \bar{b}\right) + R_{t-1}\bar{b}$$

where φ is the key fiscal coefficient, and $\bar{b}$ is the steady state interest rate. When $\varphi = 1$, the taxes adjust to exactly match the interest payment on debt. In this case, public debt is constant and equal to $\bar{b}$. When $\varphi < 1$, public debt can exhibit long lasting deviation from its steady state value. We have to impose the following inequality for the public debt to be stationary.

Condition 1 : $1 - \beta < \varphi$

The previous condition states that taxes should react enough to cover the interest payment on public debt in steady state (as shown below). As β is close to 1, the value of φ can be quite low.

In each period, the central bank creates a new quantity of money M_t^{CB}. The real quantity created $m_t^{CB} = M_t/P_t$ is used to buy a nominal quantity B_t^{CB} of asset by open market operation. Denote as M^{tot} the total nominal quantity of money. The law of motion of M^{tot} is simply $M_t^{tot} = M_{t-1}^{tot} + M_t^{CB}$, or in real terms

$$m_t^{tot} = \frac{m_{t-1}^{tot}}{1+\pi_t} + m_t^{CB} \tag{3}$$

As the asset market has been designed such that private and public assets are perfect substitute, to simplify the exposition. As a consequence, we do not differentiate between private and public asset bought by the Central Bank. As the return on all assets between period $t-1$ and period t is denoted R_{t-1}, the period t real profits are (with $b_{t-1}^{CB} = B_{t-1}^{CB}/P_{t-1}$)

$$\Theta_t = R_{t-1} b_{t-1}^{CB}$$

It is useful to denote as the difference between the amount of public debt issued by the state and the amount of financial assets bought by the Central Bank $b_t^o = b_t - b_t^{CB}$. As $m_t^{CB} = b_t^{CB}$ the consolidated budget constraint of the state is

$$b_{t-1}^o R_{t-1} + g_t = b_t^o + m_t^{CB} + \tau_t$$

Denote as γ_t the ratio of increase in the money stock in each period compared to the real money stock the previous period

$$\gamma_t = \frac{m_t^{CB}}{m_{t-1}^{tot}}$$

We wile study the model under various type of monetary policies. In the Sections below, we study a the effect of shocks to the money supply, interest rate rules, and optimal monetary policy for given process for the technology shocks. As a consequence, we will specify the process for monetary policy in each case.

3.5 Markets and equilibrium definition

There are four markets in this economy. First, the clearing of money market is $M^{tot} = \frac{\Omega M_t^n}{2}/2$, what is, in real terms

$$m^{tot} = \frac{\Omega m_t^n}{2}$$

The previous equality stipulates that only half of the non participating agents $(\Omega/2)$ hold money at the end of the period.

The financial market equilibrium is the equality of the asset demand by participating agents and the asset supply by both the firms and the State, and the central bank. The financial market budget constraint is $(1-\Omega)\,B_t^p + B_t^{CB} = B_t + P_t K_t$. Introducing, $b_t^o = b_t - b_t^{CB}$, this can be written in real term as

$$(1-\Omega)\,b_t^p = b_t^o + k_t$$

Finally the good market equilibrium is

$$(1-\Omega)\,c_t^f + \frac{\Omega c_t^H}{2} - k_t = Y_t + (1-\lambda)\,k_{t-1}$$

Given the process for the technology and for a given monetary policy, an equilibrium of this economy is a sequence of individual choices and prices $\{c_t^n, m_t^n, c_t^p, R_{t,}w_t\}$ a sequence of money stock, outstanding public debt and taxes $\{m_t^{tot}, b_t^o, \tau_t\}$ such that agents make optimal choices, the budget of the State is balanced, taxes are set according to the fiscal rule and markets clear.

4 Steady State and the linear model

4.1 Steady State

The goal of this simple model is to study monetary policy in the business cycle, for this reason, we study the dynamics around a simple steady state to obtain the simplest analytical expressions. We define our steady state as an environment were the central bank does not create money $\gamma = 0$ and were the level of public debt is 0. From equation (3), we find the net inflation rate is simply $\pi = 0$. Moreover, we assume now that the depreciation rate is $\lambda = 1$.

The real variables are easy to find in this economy. First, the real interest rate is simply

$$R^* = 1/\beta$$

One easily deduce the steady state capital stock and real wage.

$$k^* = (\alpha\beta)^{\frac{1}{1-\alpha}}$$
$$w^* = (1-\alpha)\,(k^*)^{\alpha}$$

The steady state money stock m^* and steady state taxes are

$$m^* = (1-\alpha)(k^*)^\alpha$$
$$\tau^* = 0$$

Finally one finds the steady state consumption level of each type of agents

$$c^{n*} = (1-\alpha)(k^*)^\alpha + \frac{m^*}{1+\pi} \tag{4}$$
$$c^{p*} = (R^*-1)\frac{k^*}{1-\Omega} + w^* \tag{5}$$

4.2 Linear model

We linearization of the model around the steady state is straightforward. The linearization of the Euler equation and budget constraint equation for the participating-agents yields

$$Ec^p_{t+1} - c^p_t = \frac{\alpha-1}{\sigma}k_t + \frac{1}{\sigma}Ea_{t+1} \tag{6}$$
$$k_t + (\Psi(\Omega)-1)c^p_t = \Psi(\Omega)(\alpha k_{t-1} + R^* a_t) + \frac{1}{k}m^{CB}_t + \Omega\varphi\frac{R^*}{k^*}b^p_{t-1} \tag{7}$$

Where the variable Ψ is defined as

$$\Psi(\Omega) \equiv (\alpha + (1-\Omega)(1-\alpha))\frac{R}{\alpha}$$

The parameter Ψ plays an important role in the model. It captures the effect of limited participation structure on the budget constraint of participating agents, who hold the capital stock. First, it affects the right hand side of equation (7). Indeed, the term $\Psi(\alpha k_{t-1} + R^* a_t)$ is the share of the wage bill which goes to participating agents. As non participating agents also receive some wages, participating agents (who hold the capita stock) receive only a fraction of total wage bill.

Second, as participating agents hold the capital stock K_t the *per capita* capital stock is $K_t/(1-\Omega)$. As a consequence, the when the number of participating agents decrease, (Ω increases), each participating agents has a higher financial wealth, everything else been equal.

This may affect its savings decision. This second effect appear at the left hand side of the linearized budget constraint, through the term $(\Psi - 1)\, c_t^p$.

The linearization of the other equations is straightforward.

$$
\begin{aligned}
c_t^n &= a_t + \alpha k_{t-1} - \frac{1}{w^*}\varphi R^* b_{t-1}^p - \frac{1}{\Omega m^*} m_t^{BC} \qquad (8)\\
\pi_t &= m_{t-1}^n - m_t^n + \frac{2}{\Omega m^*} m_t^{BC}\\
m_t^n &= a_t + \alpha k_{t-1} - \frac{1}{w^*}\varphi R b_{t-1}^p
\end{aligned}
$$

$$
b_t^p = (1-\varphi)\, R^* b_{t-1}^P - m_t^{CB} \qquad (9)
$$

$$
a_t = \rho^a a_{t-1} + \varepsilon_t^a
$$

The condition (9) shows that we must have $(1-\varphi)\, R^* < 1$ for the public not to diverge because taxes are less that the interest payment on debt. As $R^* = 1/\beta$. This is the same condition as Condition 1.

5 Optimal allocation

The optimal allocation is defined as a benchmark, to study the distortions of the market economy. We consider the following Pareto weight. The central planner gives a weight λ to participating agents and a weight 1 to non participating agents. We use the hat to indicate the optimal allocation. For instance $\hat{c}_t^n$ is the optimal consumption of a non-participating worker in period t. As a consequence, the social welfare function is

$$
\max E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\Omega}{2} u\left(\hat{c}_t^n\right) + \lambda\,(1-\Omega)\, u\left(\hat{c}_t^p\right) \right)
$$

The budget constraint of the central planner is

$$
\frac{\Omega \hat{c}_t^n}{2} + (1-\Omega)\, \hat{c}_t^p + \hat{K}_t = A_t^\alpha \hat{K}_{t-1}
$$

subject to $A_t = e^{a_t}$, where a_t follows the AR(1) process

$$
a_t = \rho a_{t-1} + \varepsilon_t^a
$$

The optimal allocation simply defines the dynamics of aggregate consumption c^{tot}, which is determined by the two equations

$$\left(\hat{c}_t^{tot}\right)^{-\sigma} = \beta E_t\left(\alpha A_{t+1}\hat{K}_t^{\alpha-1}\right)\left(\hat{c}_{t+1}^{tot}\right)^{-\sigma} \tag{10}$$

$$\hat{c}_t^{tot} + \hat{K}_t = A_t\hat{K}_{t-1}^{\alpha} \tag{11}$$

The optimal allocation of consumption across the two types of agents is simply

$$\hat{c}_t^{p} = \hat{c}_t^{tot}/\left(\frac{\Omega}{2}\lambda^{-\frac{1}{\sigma}} + 1 - \Omega\right) \tag{12}$$

$$\hat{c}_t^{n} = \lambda^{-\frac{1}{\sigma}}\hat{c}_t^{tot}/\left(\frac{\Omega}{2}\lambda^{-\frac{1}{\sigma}} + 1 - \Omega\right)$$

From this results, we can prove the following proposition.

Proposition 1 *The market and optimal allocations are the same in steady state if*

$$\lambda = \left(\frac{2\left(1-\alpha\right)\left(1-\Omega\right)}{\alpha\left(\Omega-\beta\right)+1-\Omega}\right)^{-\sigma} \tag{13}$$

Proof. In steady state ($A_t = 1$), the equation (10) implies that the steady state marginal productivity of capital is $1/\beta = \alpha\hat{K}^{\alpha-1}$. As the steady state interest rate in the market economy is $1/\beta$, it implies that the steady state marginal productivity of capital in the market economy is also $1/\beta$. As a consequence, the level of output is always optimal in the market economy. As a consequence, one finds that optimal total consumption in steady state $\hat{c}^{tot*}$ is equal to the market one: $\hat{c}^{tot*} = c^{p*} + c^{n*}$. Equalizing, c^{p*} and $\hat{c}^{p}$ (and thus c^{n*} and $\hat{c}^{n}$), given by equations (5) and (12), one finds the relevant value for λ. ■

The previous proposition allows to study the distortions of the market economies in the business cycle, without focusing on the steady state distortions. Indeed, as the paper is not on the optimal long run quantity of money, I will simply assume that the preferences of the Central Planner are such that the optimal steady state inflation rate is indeed 0. I will assume that the value of λ is indeed given by (13).

6 The effect of technology shocks

We now study the effect of technology shock in the market economy, when there is no monetary policy intervention

$$m_t^{CB} = 0$$

In this case, public debt is always equal to its steady state value. The program of participating agents is simply

$$E c_{t+1}^{p} - c_t^{p} = \frac{\alpha - 1}{\sigma} k_t + \frac{1}{\sigma} E a_{t+1} \tag{14}$$

$$k_t + (\Psi(\Omega) - 1) c_t^{p} = \Psi(\Omega)(\alpha k_{t-1} + a_t) \tag{15}$$

These to equations determine the dynamics of the two variables c_t^p and k_t independently as a function of the technology shock a_t , independently of other variables. To understand the distortions generated by limited participation it is useful to compare this program to the linearized program of the Central Planer (linearization of equations 10 and 11 using 12), which is

$$E_t \hat{c}_{t+1}^{p} - \hat{c}_t^{p} = \frac{\alpha - 1}{\sigma} \hat{k}_t + \frac{1}{\sigma} E a_{t+1}$$

$$\hat{k}_t + \left(\frac{R}{\alpha} - 1\right) \hat{c}_t^{f} = \frac{R^*}{\alpha}\left(\alpha \hat{k}_{t-1} + a_t\right)$$

Comparing the equations above, one first observes that they are the same when $\Psi(\Omega) = \frac{R}{\alpha}$. This is the case when $\Omega = 0$, that it when all agents participate in financial markets, as $\Psi(0) = R^*/\alpha$ As a consequence, one finds that the market allocation is optimal when all agents participate. This first shows that the assumptions about the structure of financial markets does not introduce any distortion in the economy, but just simplify the algebra.

Second, this shows that all distortions of the market economy are captured by the difference between $\Psi(\Omega)$ and $\frac{R}{\alpha}$. The previous equations show that these distortions will affect both the consumption of participating agents and the dynamics of the capital stock. In other words, both allocative and productive efficiency will be affected by these distortions.

The solution of the market dynamics (14)-(15) is simple to derive. From the capital stock dynamics one finds the consumption of the non-participating agents $c_t^n = a_t + \alpha k_{t-1}$ from equation (8). This analysis is done in Appendix, the main results are summarized in the next twos proposition. The first proposition considers productive efficiency. The second one is about allocative efficiency.

Proposition 2 *Effect of a technology shock. Assume that* $m_t^{CB} = 0$*, then*
1) the dynamics of capital in the market economy is

$$k_t = B k_{t-1} + D^a a_t \text{ with } B, D^a > 0.$$

The optimal dynamics of the economy is

$$\tilde{k}_t = \tilde{B}\tilde{k}_{t-1} + \tilde{D}^a a_t \text{ with } B, D^a > 0$$

2) If $\sigma = 1$ then the market and the optimal allocation are the same $k_t = \tilde{k}_t$. Moreover, $B = \alpha$ and $D^a = 1$

3) $\sigma > 1$ and $\Omega > 0$, then there exists a threshold ρ^s such that

- *If $\rho^a < \rho^s$, $D^{opt} > D^a$.the economy under react to a technology shock on impact*
- *If $\rho^a > \rho^s$, $D^{opt} < D^a$ the economy invest too much after a techno shock.*
- *The persistence of techno shock is lower than the optimal persistence.*

The first part of the proposition shows that the dynamics of the capital stock is simple in both the market economy and for the optimal allocation. Both processes are simple AR(1). The second part compares the dynamics. The results consider two cases.

Either $\sigma = 1$, or $\sigma > 1$. The case $\sigma = 1$ is a special case, where market and optimal allocations are the same, whatever the fraction of people participating to financial markets. Moreover, the reaction of the economy to technology shock does not depend on its persistence, ρ. It is known that the when $\sigma = 1$ both income and substitution effect balance each other such that there is no wealth effect. As the income of participating agents is equal to its optimal level in steady state, the equilibrium fluctuations of the capital stock are optimal. The case $\sigma = 1$ can be considered as a special case, where expected wealth does not affect the business cycle. Following, the business cycle literature, we consider that the case $\sigma = 1$ is the most relevant.

The third item of the proposition shows that the direction of the distortion of market economies depends on the persistence of the technology shock. For a low persistence, the market economy does not invest enough after a technology shock. For a high persistence, the market economy invest too much. This outcome is the result of the wealth effect generated by the technology shock. For a low technology shock participating agents experience a low expected increase in wealth. The opposite occur when the persistence of the technology shock. Indeed too effects compete, as can be seen in equation (15). First participating agents receive only a fraction of labor income (the $\Psi(\Omega)$) at the right hand side. But they have a higher wealth per capita (the $\Psi(\Omega) - 1$) at the left hand side.

The next Proposition presents results for the allocation of consumption for the two agent types.

Proposition 3 *If ρ close to* 0, c_t^p *increases less than* $\tilde{c}_t^p$ *on impact and* c_t^n *increases more than* $\tilde{c}_t^n$.
If ρ close to 1 c_t^p *increases less than* $\tilde{c}_t^p$ *on impact,* c_t^n *increases more than* $\tilde{c}_t^n$.

If the persistence of the technology shock is low, the participating agent consume too much on impact and non-participating agents consume too little (compared to first best). When persistence is high the reverse occurs. To analyze how monetary policy can correct the misallocation of market economies, we first independently study the effect of monetary shock.

Example 1 : As an illustrative example, the next graph presents the result of a numerical simulation. We take $\beta = 0.99, \sigma = 4, \alpha = 0.36, \Omega = 0.6$ and $\rho^a = 0.0$ and $\sigma^a = 0.01$. Technological shock are thus IID.

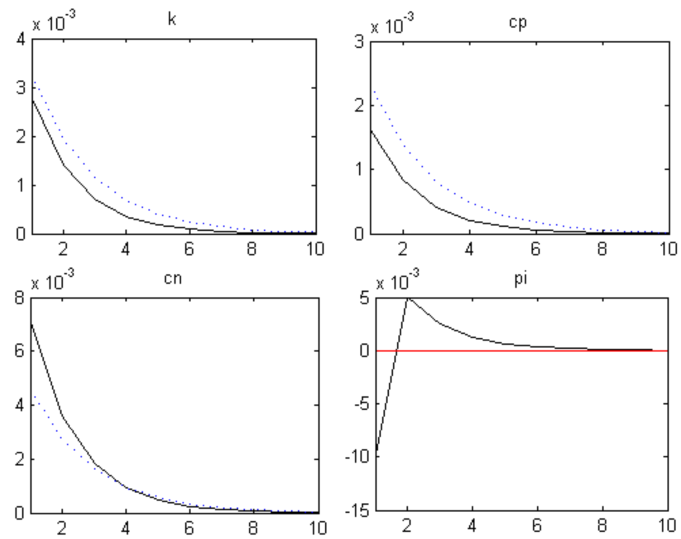


Fig. 1 : Effect of a transitory technology shock. The solid line is the market economy. The dashed is the optimal allocation.

One observes that the economy underinvest and that the consumption of participating agent is too low and the consumption of non-participating agent is too high.

Example 2: The economy as the same calibration as before except that, it is now assumed that $\rho^a = 0.95$.

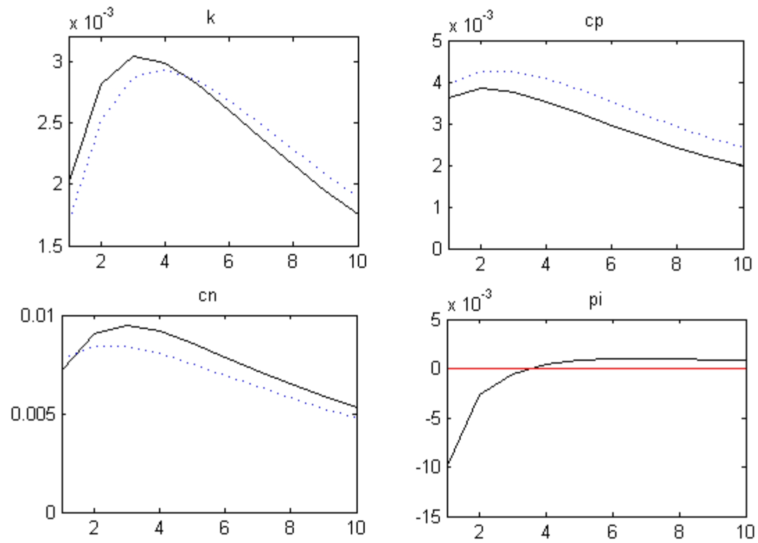


Fig. 2 : Effect of a persistent technology shock. The solid line is the market economy. The dashed is the optimal allocation.

Investment is now too high. The consumption of participating agents is too low and the consumption of non-participating agents is too high on impact, although it becomes rapidly lower that the first best value.

6.1 Effect of a monetary policy shock

In this Section, it is assume that the quantity of money follows the following process

$$m_t^{CB} = \rho^{CB} m_{t-1}^{CB} + \varepsilon_t^m$$

where ρ^{CB} is the autocorrelation of money creation and ε_t^m is $\mathcal{N}(0, \sigma^m)$. One can easily find the solution of the system (6) - (9). The next proposition summarizes the dynamics.

Proposition 4 *If the economy is hit by monetary policy shocks then the dynamics of the economy is*

$$k_t = Bk_{t-1} + C^m m_t^{CB} + \Omega D^m b_{t-1}^p$$
$$b_t^p = (1-\varphi) R^* b_{t-1}^P - m_t^{CB}$$

$B, C^m, D^m > 0$.

The previous Proposition summarizes the dynamics of capital accumulation after a monetary shock. First on impact, as money is created by open market operation, the Central Bank buys some asset on financial market what contributed to increase the demand for asset and favor capital accumulation. This increases wages, what sustain a further capital accumulation at a rate B. But the profit generated by the central banks re redistributed to the state, what contributed to a decrease in taxes. But as participating agents hold all the stock of public debt and only partially benefit from the decrease in taxes (because non-participating agents also benefit), the decrease in taxes has an overall negative effect on capital accumulation. In other words, Ricardian equivalence does not hold because some agents are not participating. This effect disappears when $\Omega = 0$ (all agents participate).

The next proposition presents the result for consumption.

Proposition 5 *After a positive monetary shock, inflation increases, c_t^n decreases. If the utility function is not too concave $\varepsilon < \varepsilon^s$, c_t^p increases.*

Money creation is a tax on money holders because it increases inflation. As a consequence, the non participating agents have a lower income and consume less. The effect on participating agents is ambiguous and depends on the strength of the substitution and income effect. It can be shown that if the utility function is not too concave, then the consumption of participating agents increases. To summarize, money shock favors capital accumulation and increase wages, but it favors mainly participating agents who do not pay the inflation tax. This result provides some intuitions on how monetary policy could decrease the distortions of the market economy. The optimal monetary policy is derived below. Before, I derive additional results about monetary policy in this environment.

6.2 Liquidity effect

Participation models have first been developed to rationalize the short-run decline in nominal interest rate after a money injection. At the first order the nominal interest rate denoted as i_t is

$$i_t = r_t + E_t \pi_{t+1} \tag{16}$$

In participation models, two effects are at stake. The first one is the decrease in the real interest rate. The second one is the increase in the inflation rate. Indeed, open market operation increases the demand for assets what decreases the real interest rate and favors capital accumulation. Second, a money shock generates anticipation of higher money creation in the future what creates some inflation and increases the nominal interest rate after the monetary shock. The gives the condition under which a liquidity effect occurs in our model.

Proposition 6 *After a money shock, the real interest rate decreases, inflation expectations increases. The nominal interest rate decreases if*

$$2\rho^m < \Omega\varphi R + \Omega w F$$

where F is the positive constant

$$F = \frac{1}{k}\frac{1-\rho^m + \Omega\varphi R\frac{\Psi-1}{\Psi-(1-\varphi)R}}{\Psi - \rho^m} > 0$$

First, if all agents participate to financial market ($\Omega = 0$) then there can be any liquidity effects. Second, if money shocks are uncorrelated $\rho^m = 0$ then there is always a liquidity effects, as soon as there is limited participation $\Omega = 0$. Third, in the general case, the existence of liquidity effect depends on the parameter of the model. The reason is the following, as money creation favors capital accumulation, it increases next period real wage and thus money demand. This effect tends to mitigate the inflationary effect of money creation. The coefficient F summarizes all these effects.

Example 3 : The effect of a monetary shock. We take the same calibration as in Example 1, assuming that there is no technology shock, $\varepsilon^a = 0$. We assume $\rho^m = 0.2$ and $\sigma^m = 0.01$.

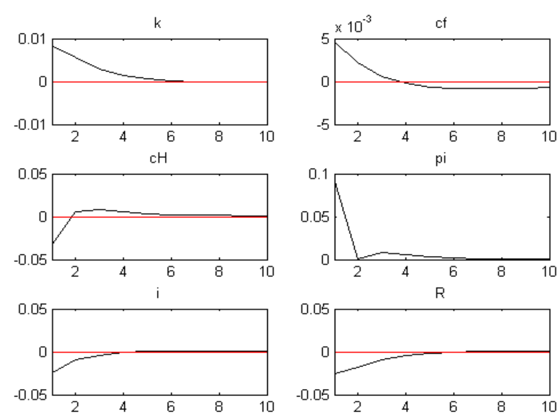


Fig. 3 : Effect of a money supply shock ..

One can observe that the economy exhibit a persistent liquidity effect.

6.3 Interest rate rule and price determinacy

We now analyze interest rate rules to investigate the effects of monetary policy shocks as unexpected change in nominal interest rates. We assume that the nominal interest rate is

$$i_t = a_1 E_t \pi_{t+1} + a_2 m_t + \varepsilon_t^i \tag{17}$$

The model is now composed of the previous equations (6) - (9) plus the definition of the nominal interest rate (16) and the monetary policy rule (17). The focus of this Section is first on price determinacy in a limited participation model. Indeed, it is know that in cashless economy a necessary condition for price determinacy is $a_1 > 1$. This results is obtained in flexible and sticky-price models without heterogeneity in money holdings. We show here that his results does not hold if the Central Bank consider the quantity of money:

The mode with an interest rate rule is not as direct to solve as the previous ones. After some algebra, one finds that the dynamics of the economy can be summarized by the four following equations in the four variables $(k_t, \pi_t, b_t^p, m_t^{CB})$

$$E_t k_{t+1} - \frac{1}{\sigma}\left(\left(1-\alpha\left(1-\sigma\right)\right)\Psi + \alpha + \sigma - 1\right)k_t + \Psi\alpha k_{t-1} + \frac{1-\rho^{CB}}{k^*} m_t^{CB} - \Omega\varphi\frac{R^*}{k^*}\left(b_t^p - b_{t-1}^p\right) = 0$$
$$\left(a_1 - 1\right)E_t\pi_{t+1} + a_2\left(\alpha k_{t-1} - \frac{1}{w^*}\varphi R^* b_{t-1}^p\right) - \left(\alpha - 1\right)k_t + \varepsilon_t = 0$$
$$E_t\pi_{t+1} = \alpha k_{t-1} - \frac{1}{w^*}\varphi R b_{t-1}^p - \left(\alpha k_t - \frac{1}{w^*}\varphi R b_t^p\right) + \frac{2}{\Omega m^*} E_t m_{t+1}^{BC}$$
$$b_t^p = \left(1-\varphi\right) R^* b_{t-1}^P - m_t^{CB}$$

The condition for determinacy can be found analytically. It amounts to check that the Blanchard-

Kahn conditions are fulfilled. The expressions are not illuminating. Instead, we present a calibrated economy to study the values of a_1 and a_2 for which the price level is determinate. We consider the same calibrated variables as in Example 1.

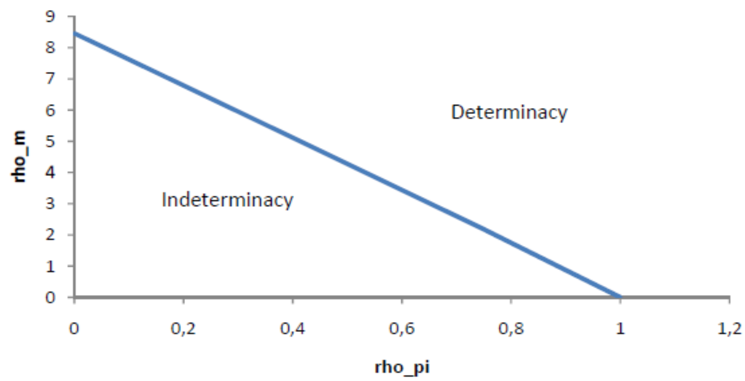


Fig. 4 : Determinacy frontier ..

One can see from Figure 4 that if central bank reacts to the quantity of money, price can be determinate even if $\rho^{\pi} < 1$. At one extreme, if the Central Bank does not target inflation, but the quantity of money, and reacts enough to the real monetary base, then prices are determinate.

7 Optimal Monetary Policy

We can now study optimal monetary policy in this environment. We consider that the central bank has can create some money in each period observing to the state of the economy. The central bank can commit to the optimal monetary policy and uses the quantity of money as its instruments. Accordingly, many other exercises could be performed in this environment: we could consider various simple rules using the quantity of money creation or the interest rate as an instrument. As the goal of this simple model is to identify the effects, the most direct case is here considered.

The next proposition is the main result.

Proposition 7 *The optimal monetary policy is*

$$\begin{aligned} m_t^{CB} &= \frac{\tilde{B} - B}{C^m} k_{t-1} + \left(\frac{\tilde{D}^a - D^a}{C^m} \right) a_t - \frac{\Omega D^m}{C^m} b_{t-1}^p \\ b_t^p &= (1 - \varphi) R^* b_{t-1}^P - m_t^{CB} \end{aligned} \tag{18}$$

in this case, the market allocation is the optimal allocation.

Proof. The proof is simple. As the model is linear, the behavior of the capital stock is

$$\begin{aligned} k_t &= B k_{t-1} + D^a a_t + C^m m_t^{CB} + \Omega D^m b_{t-1}^p \\ b_t^p &= (1 - \varphi) R^* b_{t-1}^P - m_t^{CB} \end{aligned}$$

The optimal dynamics of the economy is

$$\tilde{k}_t = \tilde{B}\tilde{k}_{t-1} + \tilde{D}^a a_t \text{ with } B, D^a > 0$$

As a consequence, if the condition (18) is fulfilled, then the level of the capital stock is optimal. We can now show that in this case, consumption levels are optimal. This can be seen from the linearization of (1), one finds

$$Ec^p_{t+1} - c^p_t = \frac{\alpha - 1}{\sigma} k_t$$

As a consequence, if the capital stock is optimal the expected growth rate of consumption for optimal (one can check that it is the same when $\Omega = 0$). As a consequence, the dynamics of consumption of participation agents is the optimal. Due to the goods market equilibrium, the consumption level of non-participating agents is optimal. ■

One can derive the properties of the optimal monetary policy with Propositions (2) - (5). When the persistence of technology shock is low, more precisely when it is below the threshold defined in , the economy underinvest after a positive technology shock. A monetary injection can restore the optimal level of investment. In other words, the optimal monetary policy is procyclical.

When the persistence of technology is high, then the market economy accumulate too much capital after a positive technology shock. Monetary policy decreases the quantity of money for investment to decrease after a positive monetary shock. Optimal monetary policy is countercyclical.

Fig. 5 plots the optimal monetary policy after a persistent technology shock, represented by the dynamics of a. The capital stock k is plotted, it is equal to its optimal value. The inflation rate is first negative before becoming positive. It implies that the return on money first increases before decreasing. The quantity of money decreases on impact before increasing. The interest rate increases but exhibits non-monotone dynamics. Finally, the public outstanding debt first increases, and then increases because the quantity of money created increases.

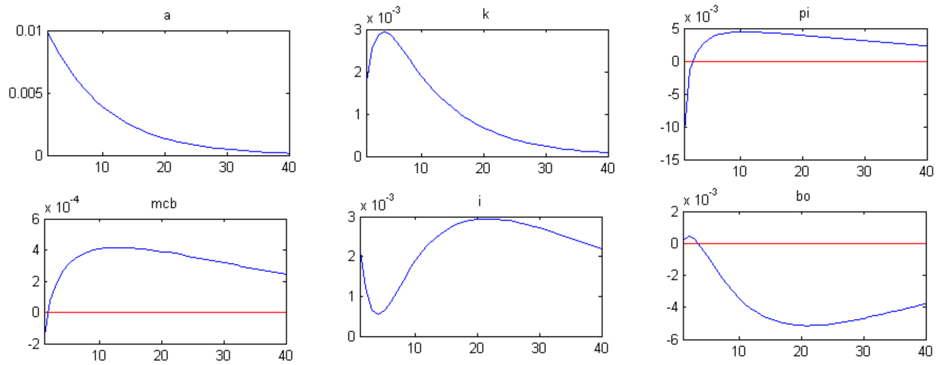


Fig. 5 : Optimal monetary policy after a persistent technology shock ..

8 The general model

A more general model is now presented, to consider more general money distribution. Indeed, in the data, households who participate in financial markers also hold money. Moreover, it is useful to introduce a more flexible framework to provide a simple calibration of the effects.

First, following the Bewley tradition, it is assumed that households face uninsurable stochastic income shocks, which motivate a money demand. Second, following the Baumol-Tobin tradition, it is assumed that households do not always participate in financial markets. More precisely, two forms of limited participation is introduced to be consistent with the data. First, a fraction of agents never participate in financial markets and only use money to smooth consumption, as in the original Bewley model. Second, participating agents randomly participate in financial markets an may demand money to consume when they do not participate in financial markets as in the Baumol-Tobin literature.

Incomplete market and limited participation models are know to be difficult to analyze with aggregate shocks. To capture the essence of limited participation and incomplete markets, I elaborate on Lucas (1990) to introduce limited insurance within two types of families. To investigate the effect of heterogeneity, I do not assumed that all agents meet within the family at the end of each period, but it is assumed that some only agents stay outside the family, what generates time-varying precautionary saving.

8.1 Households

There is a unit mass of households. A fraction Ω of households does not participate in financial markets, and is denoted non-participating households. The remaining fraction $1-\Omega$ participates, and is denoted participating households.

All households face a uninsurable employment risk. When there are employed the probability to stay employed is α^a. When they are unemployed the probability to stay unemployed is ρ. As a consequence, the number of employed agents in all periods is

$$n^e = \frac{1-\rho}{2-\alpha-\rho}$$

In all period, there is thus a fraction n^e of employed and a fraction $1-n^e$ of unemployed households among both participating and non-participating agents.

8.1.1 Non-participating households

It is assumed that non-participating agents belong to a family. The family has two locations. Employed agents live on an island, where there is full risk sharing among non-participating employed agents. Unemployed agents leave on an island, where there is full risk sharing among non-participating unemployed agents. Agents who loose their job (with a probability $1-\alpha$) travel from the employed to the unemployed island at the end of the period, after the consumption saving choice is made. Agents finding a job (with a probability $1-\rho$) travel from the unemployed to the employed island at the end of the period, after the consumption saving choice is made in both island. All households traveling across island can take its money with him.

All agents in the non-participating employed island supply one unit of labor and earn a real wage w_t. All households on the non-participating unemployed island have a home production δ. Finally, all nonparticipating households can travel to the money market and pay taxes τ_t to the state.

The timing is the following. First, family head in pool the resources. Second the aggregate shocks is revealed, which is technology and the money market shock. Third, the consumption-saving choice is made. Fourth, agents idiosyncratic choice is revealed, and households travel across islands carrying their money with them.

Denote as m_t^e the per capita real money holdings of non-participating employed households at the beginning of the period. There is a fraction n^e of such agents. Similarly, denote as m_t^u the beginning-of-period per capita money holdings of non-participating unemployed households. There is a fraction $1-n^e$ of such households. Denote as $V^e\left(m_t^e\right)$ the intertemporal welfare of an employed households and $V^u\left(m_t^u\right)$ the intertemporal welfare of an unemployed households.

The program of the family head in the employed island is to maximize the utility of all employed households taking into account the consequence of the choices for all family member.

The family head chooses per capita consumption c_t^e and per capita money holdings $\tilde{m}_{t+1}^e$ to solve.

$$\max_{c_t^e, m_{t+1}^e} n^e u\left(c_t^e\right) + \beta E_t \left[n^e V^e\left(m_{t+1}^e\right) + \left(1 - n^e\right) V^u\left(m_{t+1}^u\right)\right]$$

subject to per capita budget constraint

$$c_t^e + \tilde{m}_{t+1}^e = \frac{m_t^e}{1 + \pi_t} + w_t - \tau_t$$
$$\tilde{m}_{t+1}^e \geq 0$$

The quantity $\tilde{m}_{t+1}^e$ is the quantity of money held by employed agents before some of them travel. As a consequence, it is different from the next period quantity of money m_{t+1}^e to be determined below.

Following the same steps, the family head of the unemployed agents chooses per capita consumption c_t^u and per capita money holdings $\tilde{m}_{t+1}^u$ to solve.

$$\max_{c_t^u, m_{t+1}^u} \left(1 - n^e\right) u\left(c_t^u\right) + \beta E_t \left[n^e V^e\left(m_{t+1}^e\right) + \left(1 - n^e\right) V^u\left(m_{t+1}^u\right)\right]$$

under the per capita budget constraint

$$c_t^u + \tilde{m}_{t+1}^u = \frac{m_t^u}{1 + \pi_t} + \delta - \tau_t$$
$$\tilde{m}_{t+1}^u \geq 0$$

Writing the value functions $V^e\left(m_t^e\right)$ and $V^u\left(m_t^u\right)$ one easily finds the envelop conditions

$$V_1^e\left(m_t^e\right) = u'\left(c_t^e\right)$$
$$V_1^u\left(m_t^u\right) = u'\left(c_t^u\right)$$

We construct the equilibrium by a guess-and-verify strategy. Assume that employed agents hold a positive quantity of money ($\tilde{m}_{t+1}^e > 0$) and that unemployed agents decide to hold no money ($\tilde{m}_{t+1}^u = 0$). In this case, the quantity beginning-of-period quantity of money of households is only the money of previously employed households. As a consequence, in per capita terms, $m_{t+1}^e = \left(\alpha n^e\right) \tilde{m}_{t+1}^e / n^e$ and $m_{t+1}^u = \left(1 - \alpha^e\right) n^e \tilde{m}_{t+1}^e / \left(1 - n^e\right)$, or after some algebra

$$m_{t+1}^e = \alpha^e \tilde{m}_{t+1}^e$$
$$m_{t+1}^u = \frac{1 - \rho^e}{\alpha^e} m_{t+1}^e$$

Solving for the first order condition for the head of employed agents and the two envelop conditions, one finds the Euler equation

$$u'\left(c_t^e\right) = \beta E_t\left[\alpha^e u'\left(c_{t+1}^e\right) + \left(1-\alpha^e\right)u'\left(c_{t+1}^u\right)\right]\frac{1}{1+\pi_{t+1}}$$

As unemployed agents do not save, we have the following two budget constraints

$$\begin{aligned} c_t^e + \frac{1}{\alpha^e}\tilde{m}_{t+1}^e &= \frac{m_t^e}{1+\pi_t} + w_t - \tau_t \\ c_t^u &= \frac{1}{1+\pi_t}\frac{1-\rho^e}{\alpha^e}m_t^e + \delta - \tau_t \end{aligned}$$

Finally, the condition for unemployed agents to hold no money can be derived from the first order condition of the family head on the unemployed island. One finds

$$u'\left(c_t^u\right) > \beta E_t\left[\left(1-\rho^e\right)u'\left(c_{t+1}^e\right) + \rho^e u'\left(c_{t+1}^u\right)\right]\frac{1}{1+\pi_{t+1}}$$

8.1.2 Participating households

Participating agents who are employed are on the employed island where the representative of the family head allocates consumption and money across employed agents. The representative also participates in financial markets and choose how much to invest per capita $\tilde{b}_{t+1}^p$ in financial markets. The representative also chooses how much money each employed participating households hold $\tilde{m}_{t+1}^p$ before idiosyncratic shocks. As before, all employed participating agents received the labor income w_t. In addition the representative of the family head receives the profits of the firms, which is χ_t and $\chi_t/((1-\Omega)\,n^e)$ in per capita terms.

It is assumed that participating employed agents falling into unemployment can only bring their money when they leave the employed island. As a consequence, they can only use money to bring resources in the unemployed island. As a consequence, the representative of the participating family head manage financial assets knowing it only provides resources to the fraction n^e of participating agents.

Denote now as $V^p\left(b_t^p, m_t^p\right)$ as the intertemporal welfare of a participating employed households and $V^{pu}\left(m_t^{pu}\right)$ the one of a participating unemployed household. The representative of the family head solves

$$\max_{\tilde{b}_{t+1}^p, \tilde{m}_{t+1}^p, c_t^p} n^e u\left(c_t^p\right) + \beta E_t\left[n^e V^p\left(b_{t+1}^p, m_{t+1}^p\right) + \left(1-n^e\right)V^{pu}\left(m_{t+1}^{pu}\right)\right]$$

the per capita budget constraint is

$$c_t^p + \tilde{b}_{t+1}^p + \tilde{m}_{t+1}^p = n^e w_t + (1-n^e)\,\delta - \tau_t + b_t^p\,(1+r_t) + \frac{m_t^P}{1+\pi_t} + \frac{\chi_t}{(1-\Omega)\,n^e}$$

$$\tilde{m}_{t+1}^p, \tilde{b}_{t+1}^p \geq 0$$

The representative of the family head on the unemployed island solves

$$\max_{,\tilde{m}_{t+1}^{pu}, c_t^{pu}} (1-n^e)\,u\left(c_t^{pu}\right) + \beta E_t\left[n^e V^p\left(b_{t+1}^p, m_{t+1}^p\right) + (1-n^e)\,V^{pu}\left(m_{t+1}^{pu}\right)\right]$$

$$\tilde{m}_{t+1}^{pu} + c_t^{pu} = \delta - \tau_t + \frac{m_t^{pu}}{1+\pi_t}$$

$$\tilde{m}_{t+1}^{pu} \geq 0$$

Writing the value functions $V^e\left(m_t^e\right)$ and $V^u\left(m_t^u\right)$ one easily finds the envelop conditions

$$V_1^p\left(b_t^p, m_t^p\right) = (1+r_t)\,u'\left(c_t^e\right)$$

$$V_2^p\left(b_t^p, m_t^p\right) = \frac{u'\left(c_t^e\right)}{1+\pi_t}$$

$$V_1^{pu}\left(m_t^{pu}\right) = \frac{u'\left(c_t^{pu}\right)}{1+\pi_t}$$

As before, adopting a guess-and-verify strategy, it is assumed that $\tilde{m}_{t+1}^{pu} = 0$. One can deduce $(1-n^e)\,m_{t+1}^{pu} = (1-\alpha^e)\,n^e\tilde{m}_{t+1}^p$ and $n^e m_{t+1}^p = \alpha^e n^e \tilde{m}_{t+1}^p$ and $b_{t+1}^p = \tilde{b}_{t+1}^p$, what can be written

$$m_{t+1}^p = \alpha^e \tilde{m}_{t+1}^p$$

$$m_{t+1}^{pu} = (1-\alpha^e)\,\frac{n^e}{1-n^e}\tilde{m}_{t+1}^p$$

Solving for the first order condition, one finds the two Euler conditions and the two budget constraints

$$u'\left(c_t^p\right) = \beta E_t\,(1+r_{t+1})\,u'\left(c_{t+1}^p\right)$$

$$u'\left(c_t^p\right) = \beta E_t\left[\alpha^e u'\left(c_{t+1}^p\right) + (1-\alpha^e)\,u'\left(c_{t+1}^{pu}\right)\right]\frac{1}{1+\pi_{t+1}}$$

$$c_t^p + b_{t+1}^p + \frac{1}{\alpha^e}m_{t+1}^p = n^e w_t + (1-n^e)\,\delta - \tau_t + b_t^p\,(1+r_t) + \frac{m_t^P}{1+\pi_t}$$

$$c_t^{pu} = \delta - \tau_t + \frac{1}{1+\pi_t}\frac{1-\rho^e}{\alpha^e}m_t^p$$

Finally, for the participating unemployed agents to choose a zero quantity of money, we must have the condition

$$u\left(c_t^{pu}\right) > \beta E_t\left[u'\left(c_t^p\right)(1-\rho^e) + u'\left(c_t^{pu}\right)\rho^e\right]\frac{1}{1+\pi_t}$$

8.1.3 Distributional consequences

It may be useful to now summarize the distributional implication of the model. The next table presents the share of households of each type, their portfolio and consumption level:

$$\begin{array}{cccc} \text{Qtty Money} & \text{Qtty Asset} & \text{Conso} & \text{Number} \\ 0 & 0 & c_t^u & \Omega\left(1-n^e\right) \\ 0 & 0 & c_t^{pu} & \left(1-\Omega\right)\left(1-n^e\right) \\ \frac{1}{\alpha^e}m_{t+1}^e & 0 & c_t^e & \Omega n^e \\ \frac{1}{\alpha^e}m_{t+1}^p & b_{t+1}^p & c_t^p & \left(1-\Omega\right)n^e \end{array} \tag{19}$$

8.2 Production side and market equilibria

The production side and the budget of the state are as before, described in Section 3.3 and 3.4. As a consequence, the firm issues a quantity of capital k_t and the state b_t^o. As a consequence, the financial market equilibrium is

$$\left(1-\Omega\right)n^e b_{t+1}^p = b_t^o + k_t$$

The money market equilibrium is

$$m_{t+1}^{tot} = \Omega n^e m_{t+1}^e/\alpha^e + \left(1-\Omega\right)n^e m_{t+1}^p/\alpha^e$$

with the law of motion

$$m_{t+1}^{tot} = \frac{m_t^{tot}}{1+\pi_t} + m_t^{CB}$$

The goods market equilibrium is

$$\left(1-\Omega\right)\left(n^e c_t^p + \left(1-n^e\right)c_t^{pu}\right) + \Omega\left(n^e c_t^e + \left(1-n^e\right)c_t^u\right) + k_t = Y_t + \left(1-\lambda\right)k_{t-1} + (1-n^e)\delta$$

8.3 Full Model

We can write the full model

$$\begin{aligned} u'\left(c_t^e\right) &= \beta E_t\left[\alpha^e u'\left(c_{t+1}^e\right) + \left(1-\alpha^e\right)u'\left(c_{t+1}^u\right)\right]\frac{1}{1+\pi_{t+1}} \\ c_t^u &= \frac{1}{1+\pi_t}\frac{1-\rho^e}{\alpha^e}m_t^e + \delta - \tau_t \\ c_t^e + \frac{1}{\alpha^e}m_{t+1}^e &= \frac{m_t^e}{1+\pi_t} + w_t - \tau_t \end{aligned} \tag{20}$$

$$u'\left(c_t^p\right)=\beta E_t\left(1+r_{t+1}\right)u'\left(c_{t+1}^p\right)$$
$$u'\left(c_t^p\right)=\beta E_t\left[\alpha^e u'\left(c_{t+1}^p\right)+\left(1-\alpha^e\right)u'\left(c_{t+1}^{pu}\right)\right]\frac{1}{1+\pi_{t+1}}$$
$$c_t^p+b_{t+1}^p+\frac{1}{\alpha^e}m_{t+1}^p=n^e w_t+\left(1-n^e\right)\delta-\tau_t+b_t^p\left(1+r_t\right)+\frac{m_t^P}{1+\pi_t}+\frac{\chi_t}{\left(1-\Omega\right)n^e}$$
$$c_t^{pu}=\delta-\tau_t+\frac{1}{1+\pi_t}\frac{1-\rho^p}{\alpha^e}m_t^p$$

$$w_t=\left(1-\alpha\right)A_t k_{t-1}^{\alpha} \tag{21}$$
$$\left(1+r_t\right)=\alpha k_t^{\alpha-1}E_t A_{t+1}+\left(1-\lambda\right)$$
$$\chi_t=\alpha k_{t-1}^{\alpha}\varepsilon_t$$

$$\tau_t=\varphi\left(1+r_{t-1}\right)\left(b_{t-1}^o-\bar{b}+m_t^{CB}\right)+\left(1+r_{t-1}\right)\left(\bar{b}-m_t^{CB}\right)-m_t^{CB}$$
$$b_t^o=b_{t-1}^o\left(1+r_{t-1}\right)-m_t^{CB}-\tau_t$$

$$m_{t+1}^{tot}=\Omega n^e m_{t+1}^e/\alpha^e+\left(1-\Omega\right)n^e m_{t+1}^p/\alpha^e$$
$$m_{t+1}^{tot}=\frac{m_t^{tot}}{1+\pi_t}+m_t^{CB}$$
$$\left(1-\Omega\right)b_{t+1}^p=b_t^o+k_t$$
$$\left(1-\Omega\right)\left(n^e c_t^p+\left(1-n^e\right)c_t^{pu}\right)+\Omega\left(n^e c_t^e+\left(1-n^e\right)c_t^u\right)+k_t=Y_t+\left(1-\lambda\right)k_{t-1}+(1-n^e)\delta$$

with the shock structure

$$A_t=\rho^A A_{t-1}+\varepsilon_t^A \tag{22}$$

8.4 Steady state

The properties of the steady state can be easily derives. First, as before, the real interest rate is pinned down by the Euler equation of the participating agents

$$1+r=\frac{1}{\beta}$$

It determines the capital stock and the real wage.

The steady ratio of consumption across agents cis

$$\frac{c^u}{c^e}=\frac{c^{pu}}{c^p}=\left(\frac{\frac{1+\pi}{\beta}-\alpha^e}{1-\alpha^e}\right)^{-\frac{1}{\sigma}}$$

One observe that the level of inflation $\pi > \beta$ generates a partial insurance against consumption risk. $c^u < c^e$ and $c^{pu} < c^p$. In the special case where $\pi = \beta$ (the Friedman Rule). all agents are perfectly insured .Using these expressions and the budgets constraints one can find all steady state values.

8.5 First Best

As in the previous Section, it is assumed that the Social planner has the following social welfare function

$$\sum_{t=0}^{\infty} \beta^t \left[\Omega \left(n^e \omega^e u\left(c_t^e\right) + n^u \omega^u u\left(c_t^u\right) \right) + (1-\Omega) \left(n^e \omega^p u\left(c_t^p\right) + n^u \omega^{pu} u\left(c_t^{pu}\right) \right) \right]$$

The social planner gives a specific weight to the consumption of each specific group.

The first best allocation is

$$\max \sum_{t=0}^{\infty} \beta^t \left[\Omega \left(n^e \omega^e u\left(c_t^e\right) + n^u \omega^u u\left(c_t^u\right) \right) + (1-\Omega) \left(n^e \omega^p u\left(c_t^p\right) + n^u \omega^{pu} u\left(c_t^{pu}\right) \right) \right]$$

$$k_t + C_t = A_t k_{t-1}^{\alpha} \left(n^e\right)^{1-\alpha} + (1-\eta) k_{t-1}$$
$$C_t = \Omega \left(n^e c_t^e + n^u c_t^u \right) + (1-\Omega) \left(n^e c_t^p + n^u c_t^{pu} \right)$$

Proposition 8 *Denote as c^p, c^e, c^u, c^{pu} the steady state consumption of the market economy with an equilibrium inflation rate π. If*

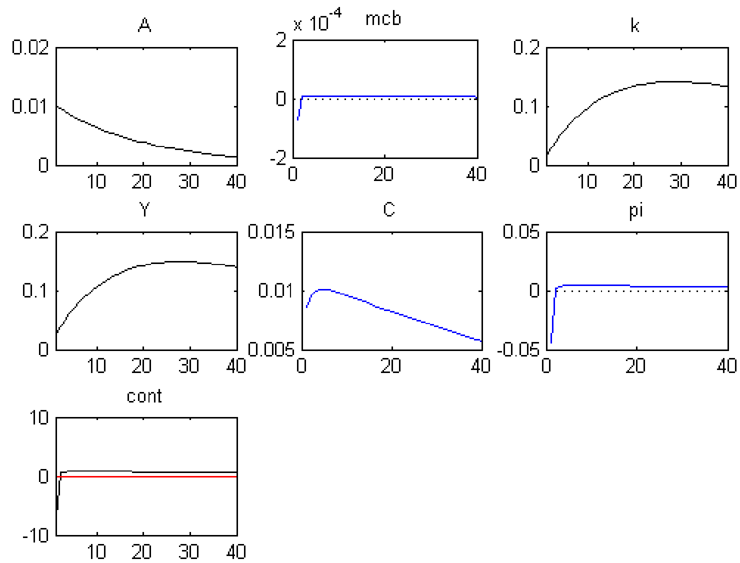
$$\frac{\omega^e}{\omega^p} = \frac{1-\Omega}{\Omega} \left(\frac{c^p}{c^e}\right)^{\sigma}, \frac{\omega^u}{\omega^p} = \frac{(1-\Omega)\, n^e}{\Omega n^u} \left(\frac{c^p}{c^u}\right)^{\sigma}, \frac{\omega^{pu}}{\omega^p} = \frac{n^e}{n^u} \left(\frac{c^p}{c^{pu}}\right)^{\sigma}$$

and $\omega^e + \omega^u + \omega^p + \omega^{pu} = 1$, the steady state market equilibrium and the steady state optimal allocation are the same.

The previous proposition exhibits a condition under which the market and optimal allocation are the same in the steady state. We will assume that the conditions of this proposition are fulfilled to only consider deviation in the business-cycle.

8.6 Calibration and simulation

We now derive the optimal monetary policy in this model, with the following calibration. The period is a quarter $\beta = 0.99, \sigma = 3$. The production function is such that $\alpha = .3$.and $\lambda = 0.025$. it is assumed that the idiosyncratic risk is the labor market risk, what yields $\alpha^e = 0.95$ and



$\rho^e = 0.22$. The annual steady state inflation rate is 2% and the persistence of the technology shock is $\rho^A = 0.95$.

The last parameter is home production. We calibrate it such that the total quantity of money over households income is 8%, which is the value found in the SCF survey. It gives $\delta = 1.14$ and a replacement ratio of 65%, what is consistent with the data.

One can check that the existence conditions are fulfilled.

The following problem is now solved

$$\max_{m_t^{cb}} \sum_{t=0}^{\infty} \beta^t \left[\Omega \left(n^e \omega^e u \left(c_t^e \right) + n^u \omega^u u \left(c_t^u \right) \right) + (1 - \Omega) \left(n^e \omega^p u \left(c_t^p \right) + n^u \omega^{pu} u \left(c_t^{pu} \right) \right) \right]$$

subject to equations from (20) to (22). A linear-quadratic approach is used. The next graph shows that a active monetary policy maximize intertemporal welfare.

9 Conclusion

This paper presents a simple theory of money demand based on recent empirical work on money demand. Some agents do not participate in financial markets and smooth consumption expenditures with money. Other agents participate to financial markets, hold both public debt and the capital stock. Money creation is modeled by open market operations. Compared to other limited participation models, the model of this paper analyses capital accumulation and allows for an identification of optimal monetary policy. The market allocation has two types of distortions. First, capital investment is not optimal after a technology shock, due to limited participation. Second, consumption inequalities are not optimal due to the difference in the return of the two assets (money and financial assets). It is found that monetary policy can restore the first best allocation for both investment and consumption.

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A Summary of the Simple Model

This Appendix presents the full model before linearization.

$$m_t = w_t - \tau_t$$
$$c_t^n + w_t + \frac{m_{t-1}}{1+\pi_t} - \tau_t$$

$$u'(c^p) = \beta R_t E_t u'\left(c_{t+1}^p\right)$$
$$b_t^p + c_t^p = R_{t-1} b_{t-1}^p + w_t + \frac{\chi_t}{1-\Omega} - \tau_t$$

$$w_t = (1-\alpha) A_t k_{t-1}^{\alpha}$$
$$R_t = \alpha k_t^{\alpha-1} E_t A_{t+1}$$
$$\chi_t = \alpha k_{t-1}^{\alpha} \varepsilon_t$$

$$\tau_t = \varphi R_{t-1}\left(b_{t-1}^o - \bar{b}\right) + R_{t-1}\bar{b}$$
$$b_t^o = b_{t-1}^o R_{t-1} - m_t^{CB} - \tau_t$$

$$m_t = \frac{m_{t-1}}{1+\pi_t} + \frac{2}{\Omega} m_t^{BC}$$
$$(1-\Omega) b_t^p = b_t^o + k_t$$
$$(1-\Omega) c_t^p + \frac{\Omega}{2} c_t^n = Y_t - k_t + (1-\delta) k_{t-1}$$
$$A_t = e^{a_t}, \text{where } a_t^a = \rho^a a_{t-1} + \varepsilon_t^a$$

B Proof of Propositions 2 and 3

B.1 Market allocation

The solution of the system (14) - (15) can be found using the method of undetermined coefficients. Assume that the market capital stock has the following form $k_t = Bk_{t-1} + D^a a_t$. Using the two equations one finds the following coefficients. I write them as explicit function of σ, Ψ, ρ to simplify the analysis below.

$$\begin{aligned}
B(\sigma,\Psi) &= \frac{1}{2\sigma}\left(\left(1-\alpha\left(1-\sigma\right)\right)\Psi+\alpha+\sigma-1\right) \\
&\quad -\frac{1}{2}\sqrt{\frac{1}{\sigma^2}\left(\left(1-\alpha\left(1-\sigma\right)\right)\Psi+\alpha+\sigma-1\right)^2-4\Psi\alpha} \\
D^a(\sigma,\Psi,\rho) &= \frac{\Psi+\frac{\rho}{\sigma}\left(\Psi\left(1-\sigma\right)-1\right)}{\frac{1}{\sigma}\left(\left(1-\alpha\left(1-\sigma\right)\right)\Psi+\alpha+\sigma-1\right)-B(\sigma,\Psi)-\rho}
\end{aligned}$$

One can check that when $\sigma = 1$, one finds $B(1,\Psi) = \alpha$ and $D(1,\Psi) = 1$. $B(\sigma,\Psi)$ is increasing in σ.

Studying the variations of $D(\sigma,\Psi,\rho)$ one finds that for $\sigma > 1$, there exists a threshold ρ^s such that $D_\Psi(\sigma,\Psi,\rho) > 0$ if $\rho < \rho^s$ and $D_\Psi(\sigma,\Psi,\rho) < 0$ if $\rho > \rho^s$.

The consumption of the two types of agents after a technology shock is

$$\begin{aligned}
c_t^p &= \left[1+\frac{1}{\Psi-\rho}\left(\rho-(1-\alpha)\frac{\Psi}{\Psi-\alpha}\right)\varepsilon\right]a_t \\
&\quad +\left[\alpha+(1-\alpha)\frac{1}{2}\left(1-\frac{1}{(\Psi+\alpha)(\Psi-\alpha)}\right)\varepsilon\right]k_{t-1} \\
c_t^n &= a_t+\alpha k_{t-1}
\end{aligned}$$

B.2 Optimal allocation

The solution of the system (10) - (11) can be found following the same steps. One finds

$$k_t = \hat{B}k_{t-1}+\hat{D}^a a_t$$

with

$$\begin{aligned}
\hat{B} &= \frac{1}{2\sigma}\left(\left(1-\alpha\left(1-\sigma\right)\right)\frac{R^*}{\alpha}+\alpha+\sigma-1\right) \\
&\quad -\frac{1}{2}\sqrt{\frac{1}{\sigma^2}\left(\left(1-\alpha\left(1-\sigma\right)\right)\frac{R^*}{\alpha}+\alpha+\sigma-1\right)^2-4\frac{R^*}{\alpha}\alpha} \\
\hat{D}^a &= \frac{\frac{R^*}{\alpha}+\frac{\rho}{\sigma}\left(\frac{R^*}{\alpha}\left(1-\sigma\right)-1\right)}{\frac{1}{\sigma}\left(\left(1-\alpha\left(1-\sigma\right)\right)\frac{R^*}{\alpha}+\alpha+\sigma-1\right)-\hat{B}-\rho}
\end{aligned}$$

One can check that the expression of $\hat{B}$, B and $\hat{D}^a, D^a$ are the same when $\Psi = \frac{R^*}{\alpha}$, what is the case when $\Omega = 0$. In other words the market and optimal capital dynamics are the same when all agents participate to financial markets.

The optimal consumption dynamics is

$$\hat{c}_t^p = \frac{\frac{R^*}{\alpha} - \hat{D}^a}{\frac{R^*}{\alpha} - 1} a_t + \frac{\alpha}{R^* - \alpha} \left(R^* - \hat{B} \right) k_{t-1}$$
$$\hat{c}_t^n = \lambda^{-\frac{1}{\sigma}} \hat{c}_t^p$$

B.3 Comparison of market and optimal allocation

Comparing $\hat{B}$ and B, one finds $B < \hat{B}$. Comparing $\hat{D}^a$ and D^a one finds that $D^a < \hat{D}^a$ when ρ is close to 0 and $D^a > \hat{D}^a$ when ρ close to 1. Looking at the of derivative $D_\rho^a (\sigma, \Psi, \rho)$ one finds that there is a unique threshold. Moreover comparing the consumption dynamics, one easily finds the result of Proposition (3).

C Proof of Propositions 4 and 5

The solution of the linear model is derived by guessing that

$$k_t = B^m k_{t-1} + C^m m_t^{CB} + D^m b_{t-1}^p$$

Solving the model with this expression, one finds that $B^m = B$. Define

$$\Lambda \equiv \frac{1}{\sigma} \left(\left(1 - \alpha \left(1 - \sigma \right) \right) \Psi + \alpha + \sigma - 1 \right)$$

Then one finds

$$D^m = \Omega \varphi \frac{R^*}{k^*} \frac{(1-\varphi) R^* - 1}{B + (1-\varphi) R^* - \Lambda}$$
$$C^m = \frac{\frac{1-\rho^{CB}}{k} + \Omega \varphi \frac{R^*}{k^*} - D^m}{\Lambda - B - \rho^m}$$

One can check that $D^m, C^m > 0$. Consumption of non-participating agents is $c_t^H = \alpha k_{t-1} - \frac{1}{w} \varphi R b_{t-1}^p - \frac{1}{\Omega m} m_t^{BC}$. The consumption of participating agents is

$$c_t^p = \frac{\Psi}{\Psi - 1} \alpha k_{t-1} + \frac{1}{\Psi - 1} \left(\frac{1}{k} - C^m \right) m_t^{CB} + \frac{\Omega \varphi}{\Psi - 1} \frac{R}{k} b_{t-1}^p - \frac{1}{\Psi - 1} \left(B k_{t-1} + D^m b_{t-1}^p \right)$$

The effect of a monetary shock on impact depends on $\frac{1}{k} - C^m$. Studying the variations one finds the results of Proposition 5.

D Proof of Proposition 6

Inflation can be written as

$$\pi_t = m_{t-1} - m_t + \frac{2}{\Omega m} m_t^{BC}$$

The nominal Interest rate is

$$i_t = r_t + E_t \pi_{t+1}$$

Substituting r_t and $E_t \pi_{t+1}$ by their expressions, one finds

$$i_t = (\alpha - B)\, k_{t-1} + \left(\frac{1}{w} \varphi R^2 (1 - \varphi) - \frac{1}{w} \varphi R - D \right) b_{t-1}^p + \left(\frac{2}{\Omega w} \rho^m - \frac{1}{w} \varphi R - C \right) m_t^{CB}$$

As a consequence, the interest rate decreases on impact if

$$\frac{2}{\Omega w} \rho^m - \frac{1}{w} \varphi R - C < 0$$