

Fiscal delegation in a monetary union with decentralized public spending*

Henrique S. Basso and James Costain

Banco de España, Calle Alcalá 48, 28014 Madrid, Spain

henrique.basso@bde.es, james.costain@bde.es

July 2013

Abstract: This paper studies the effects of delegating control of sovereign debt issuance to an independent authority in a monetary union where public spending decisions are decentralized. The model assumes that no policy makers are capable of commitment to a rule. However, consistent with Rogoff (1985) and with the recent history of central banking, it assumes that an institution may be designed to have a strong preference for achieving some clear, simple, quantitative policy goal.

Following Beetsma and Bovenberg (1999), we show that in a monetary union where a single central bank interacts with many member governments, debt is excessive relative to a social planner's solution. We extend their analysis by considering the establishment of an independent fiscal authority (IFA) mandated to maintain long-run budget balance. We show that delegating sovereign debt issuance to an IFA in each member state shifts down the time path of debt, because this eliminates aspects of deficit bias inherent in democratic politics. Delegating to a single IFA at the union level lowers debt further, because common pool problems across regions' deficit choices are internalized.

The establishment of a federal government with fiscal powers over the whole monetary union would be less likely to avoid excessive deficits, because only the second mechanism mentioned above would apply. Moreover, the effective level of public services would be lower, if centralized spending decisions are less informationally efficient.

Keywords: Fiscal authority, delegation, decentralization, monetary union, sovereign debt

JEL classification: E61, E62, F41, H63

*Views expressed in this paper are those of the authors, and should not be attributed to the Banco de España or the Eurosystem. The authors are grateful for helpful comments from Javier Andrés, Beatriz de Blas, Pablo Hernández de Cos, Juan Francisco Jimeno, Bartosz Mackowiak, Albert Marcet, Javier Pérez, Giacomo Ponzetto, and seminar participants at CFE-ERCIM 2012, the Banco de España, the Bank of England, and the ECB-CFS Joint Lunch Seminar. The authors take responsibility for any errors.

1 Introduction

In the summer of 2012, when President Draghi expressed an unambiguous commitment to preserve the Eurozone, returns on peripheral European debt plummeted. After peaking at over 5%, premia on Spanish and Italian debt were both below 3% in early 2013,¹ illustrating just how easily central banks can combat speculative attacks on bonds issued in a currency they control. Since the ECB can, in principle, emit unlimited quantities of euros, it can buy any quantity of euro-denominated debt, and thereby put a floor under Eurozone sovereign bond prices. Nonetheless, further reforms are needed to make Eurozone institutions viable in the long run. While the ECB can conjure up a cap on the risk premium through monetary policy alone, this will eventually be inflationary if peripheral countries fail to balance their budgets over the longer term. Therefore a monetary mechanism to prevent speculative attacks must be accompanied by an adequate fiscal regime if it is to ensure the permanence of the euro.

Proposals for fiscal reform have revolved around two competing interpretations of “fiscal union”. Many economists and political leaders argue that it is time to consummate the founders’ vision of a strong federal Europe with a government able to transfer resources countercyclically, from economies in expansion to those in recession.² But European electorates are skeptical of placing more power in the hands of Brussels. And core member states with healthy public finances have objected to a “fiscal transfer union”, as they call it, because of the moral hazard it creates.³ Fearing that they could end up paying for the fiscal imbalances of others, these members instead advocate a “fiscal stability union”, meaning a reinforcement of the debt and deficit limits, backed by monitoring and sanctions, that constituted the Stability and Growth Pact (SGP). Unfortunately, there is no consensus about how to ensure that a rule-based framework would succeed in the future, when so many member states broke the rules in the past.

However, it is wrong to assume that “fiscal transfer union” and “fiscal stability union” are the only options for greater fiscal discipline in Europe. As a voluntary association of nation states with large governments and distinct cultural identities, the EMU is *sui generis*, and there is no reason to suppose that institutions which

¹The Economist (19 January 2013).

²See for example Financial Times (8 December 2010), The Wall Street Journal (3 June 2011) or Pisani-Ferry (2012).

³See The Economist (2 December 2010).

have succeeded elsewhere will be appropriate for Europe.⁴ While many countries suffer budgetary problems today, fiscal indiscipline may be especially problematic in a monetary union: member countries may overexploit the joint budget as a common resource; members may expect others to rescue them if they get into trouble; and market doubts about any member's solvency may spread contagiously across the union. For all these reasons, members of a monetary union need strong fiscal regimes to ensure long-run budget balance. But by itself this is not enough: beyond ensuring their own solvency, peripheral Eurozone states must also *convince core states* that their solvency is ensured. Otherwise, core members will be unwilling to help protect them from speculative attacks; and thus even if they slash deficits enough for solvency at low interest rates, they will remain unprotected against a self-fulfilling spiral to higher rates at which they become insolvent. This analysis suggests that the Eurozone requires much stronger fiscal discipline than has been the norm in other economies.

This paper considers another way to reinforce fiscal discipline in Europe: the delegation of executive control over one or more powerful fiscal instruments to an independent institution with a mandate to ensure long-run budget balance. The motivation for fiscal delegation follows from Rogoff's (1985) analysis of central bank independence: delegating discretionary control of monetary instruments can solve the problem of inflationary bias, without requiring an inflexible commitment to low inflation under all circumstances, as long as the preferences of the central bank incorporate a countervailing, anti-inflationary bias. Since the mandates of contemporary central banks are strongly focused on low inflation, and since these banks control instruments that make low inflation feasible, they have acted "conservatively" in Rogoff's sense even in the absence of formal rules relating to inflation only. Likewise, the establishment of an independent authority with a mandate for low debt or low deficits might successfully combat deficit bias, as long as it controls instruments of sufficient power to make debt control feasible. While this idea has been largely absent from the current Eurozone debate, frameworks like this have been proposed for many countries and regions, including Australia and New Zealand (Ball (1996), Gruen (1997)); the US (Blinder (1997), Seidman and Lewis (2002)); Latin America (Eichengreen, Hausmann, and von Hagen (1999)); and the European Union (von Hagen and Harden (1995); Wren-Lewis (2002); Wren-Lewis (2011); Calmfors (2003); Wyplosz (2005); Costain and de Blas (2012a)).

⁴In particular, in the short and medium run, we doubt that the US experience is informative for the EMU, for two main reasons. First, the EMU has less consensus for political integration than the US does; in fact, the crisis may be increasing the divisions between member states. Second, member states play a much larger fiscal role in the EMU than US states do, making a policy of zero deficits at all times costlier (and less credible) for EMU members than it is for US states.

But while this idea has been widely advocated, there has been very little theoretical work to evaluate the effects of policy delegation in a fiscal context.⁵

Thus, our model studies the effects of Rogoff-style delegation of fiscal powers in the context of a monetary union. Our environment follows Beetsma and Bovenberg (1999, henceforth BB99), who study the interaction of a single central bank with many national (or regional) governments. Like Rogoff (1985), the BB99 paper posits a time-inconsistency problem like that of Barro and Gordon (1983): a surprise increase in inflation stimulates output, giving the central bank an incentive to choose an inflation rate higher than the public expects. Inside a monetary union, this inflation bias also gives rise to deficit bias by way of a common pool problem: while governments know that the central bank will be tempted to inflate away the public debt, each individual national government ignores the impact of its own debt, since this is only a small part of the total. Therefore, BB99 conclude that welfare is improved if fiscal policy is bound by rules limiting debt or deficits. But this analysis is built around a strange inconsistency: it is unclear why they take for granted that the central bank must act under discretion while assuming that the government's fiscal decisions can (and advocating that they should) be bound by a commitment to rules.⁶

Our paper strives for a more consistent treatment of rules versus discretion. We assume that no policy makers can commit— that is, given the risk of unforeseen future contingencies, no institution can irreversibly oblige itself to follow a rule. However, consistent with Rogoff (1985) and with the recent history of central banking, we assume that an institution may be designed to have a strong preference for achieving some clear, simple, quantitative goal. Just as contemporary central banks display a strong aversion to inflation, we consider a hypothetical independent budgetary authority with a strong aversion to debt. We compare the baseline scenario of BB99, in which the central bank has discretionary control of inflation and the government has discretionary control of all aspects of fiscal policy, with an alternative institutional arrangement in which the government controls the allocation of public spending, but an independent fiscal authority controls the emission of debt. As BB99 already showed, under the baseline scenario a monetary union with many member states exhibits excessive debt accumulation, relative to a social planner's solution. We show in addition

⁵Persson and Tabellini (1994) explore a different aspect of fiscal policy delegation. They point out that representative democracy is a form of policy delegation, and in line with Rogoff (1985), they argue that the median voter may prefer a representative with a relatively “conservative” view of capital taxation, in order to offset the problem of time inconsistency.

⁶Many papers written around the time of the introduction of the euro to provide theoretical backing for the Stability and Growth Pact simply assumed that rules, once established, would be followed. See for example Buti, Roeger, and In't Veld (2001).

that delegating control of debt issuance to an independent authority in each member state decreases debt at all points in time. This follows from the greater debt aversion and lower impatience of the fiscal authority, compared with a democratic government. Delegating instead to a single independent authority at the level of the union shifts down the time path of debt again, because a union-level authority internalizes common pool problems associated with decentralized fiscal decisions. At the same time, this alternative institutional setup maintains the advantages of subsidiarity, by leaving the allocation of public spending to be decided at the national level, where information is better and democratic legitimacy is greater.

In our baseline model, which follows BB99 closely, deficit bias in the monetary union is a side-effect of inflation bias. Arguably, inflationary bias is not really relevant under the current European monetary framework. However, other types of deficit biases, due to moral hazard or contagion, might be present. Therefore, in Section 4, we extend our model to include the possibility of contagion across governments' borrowing rates. We show that this contagion creates another form of deficit bias, also induced by a common pool problem: each government feels the full benefit when it runs a higher deficit, but the effect of higher debt on borrowing costs is shared across all members. All results on the ranking of debt across different institutional regimes obtained in the benchmark model continue to hold in the extended model with contagion.

While creating a new class of fiscal institutions might seem like an exotic remedy for the Eurozone crisis, we feel our analysis provides grounds for optimism, because it highlights a clear formula for political feasibility. Member states with precarious debt levels would benefit from a European system to prevent speculative attacks. Member states with strong finances fear providing this sort of guarantee precisely because they worry that it would allow weaker members to continue running excessive budget deficits. Therefore, a *quid pro quo* in which the ECB stops speculative attacks only for countries that have adopted a truly credible budget balance regime appears politically viable. The crucial point is that the fiscal regime must be solid enough to convince core members that moral hazard has been eliminated. It is hard to see how any kind of promise to follow a rule in all future circumstances, or how even the most savage short-term austerity program, can really convince core members that moral hazard is absent. This is why endowing an EU institution with instruments that give executive control over national debt could prove to be the key to longer-term Eurozone survival.⁷

⁷See de Blas (7 June 2012, VoxEU) for details on institutional implementation.

1.1 Related literature

Economists have long emphasized the fiscal challenges implied by joining a monetary union. Mundell (1961) argued that if a country gives up monetary independence, it needs countercyclical fiscal policy to offset the amplified effects of asymmetric demand shocks. More recent analyses focus on a more dramatic form of instability: by giving up their ability to emit currency independently, member states (like emerging economies that suffer from “original sin”) become vulnerable to speculative attacks on their sovereign debt (Eichengreen and Wyplosz (1998); De Grauwe (2012)). Moreover, this limits their ability to act as lenders of last resort for their domestic banks, so troubles in the public and banking sectors become mutually reinforcing (Bruche and Suarez (2010); Pisani-Ferry (2012)). The literature on monetary and fiscal interactions (*e.g.* Leeper (1991); Sims (2013)) also points to the fragility of monetary unions: the set of monetary and fiscal rules consistent with solvency and equilibrium determinacy is likely to be reduced by joining a monetary union (Bergin (1998); Sims (1999); Leith and Wren-Lewis (2011)). Another indication of fiscal vulnerabilities in a monetary union comes from the literature on deficit bias. While Dixit and Lambertini (2003) constructed an example in which joining a monetary union has no effect on policy outcomes if all policy makers have identical objective functions, many other authors argue that monetary union increases deficit bias when policy makers’ preferences differ in plausible ways (Beetsma and Bovenberg (1999); Buti, Roeger, and In’t Veld (2001); Beetsma and Jensen (2005); Chari and Kehoe (2007)).

Like inflation bias, deficit bias arises when policy makers are excessively impatient or have incentives to break past promises; thus it is natural to ask whether Rogoff’s (1985) proposal to combat inflation bias through policy delegation might also apply to deficit bias. Like Rogoff, we model institutional differences parsimoniously by assuming different weight parameters in their objective functions. First, we assume democratic politics makes elected policy makers impatient (relative to society). This reflects widespread findings in the political economy literature: for example, Alesina and Tabellini (1990) show how alternating parties of opposing ideology may act impatiently, while Battaglini (2011) shows how impatience may vary with the debt level, implying mean-reverting dynamics like those of our extended model of Sec. 4. Second, we model the effects of a policy mandate by placing extra weight on the mandated objective in the institution’s preferences (relative to society).⁸ For a simple and quantifiable objective, this seems

⁸Adam and Billi (2008) show, in a microfounded New Keynesian model, that the benefits of “conservative” central banking extend to an economy with endogenous fiscal policy. Their model shares some features with ours: they assume that the central bank’s preferences reflect those of society, but place additional weight on inflation stabilization; and they consider the Markov perfect equilibria of a simultaneous game between monetary and fiscal policy makers, under discretion.

reasonable; central banks mandated to achieve low inflation do indeed appear to attach great importance to this objective.⁹ Persson and Tabellini (1993) argue that central banks might achieve better macroeconomic stabilization (and higher social welfare) if they had more complex objectives in their mandates (or preferences). The tradeoff between simple and complex objectives is an interesting issue, but probably requires a deeper model of the effects of the mandate on institutions' decisions. For the current paper, which focuses on how systematic biases are affected by policy delegation, Rogoff's reduced-form approach provides useful insights.

Since the 1990s, as inflation fell and public debts grew in developed economies, many economists have suggested delegating some fiscal responsibilities to an institution independent of the government (see Debrun, Hauner, and Kumar (2009); Hagemann (2010); and Costain and de Blas (2012a) for surveys), though few of these proposals have been based on formal models. The literature distinguishes *fiscal councils*— which monitor but do not implement fiscal policy actions— from *independent fiscal authorities* (IFAs), which would make some of the fiscal decisions currently taken by the government. Fiscal councils are widespread today, and are mandated by recent European reforms,¹⁰ but IFAs remain hypothetical. Two main types of IFA have been proposed. On one hand, the IFA might set a deficit target, at the start of the annual budget cycle, which the government is bound to respect; alternatively, it might exercise executive control over some fiscal instrument with a strong budgetary impact.¹¹ Some papers (see Hagemann (2010), Sec. II.C; or Calmfors (2011), Sec. 1) take the nonexistence of IFAs today as evidence of their inviability, arguing that delegation is less appropriate for fiscal than for monetary policy since fiscal decisions are multidimensional, complex, and political. However, the validity of this claim depends on *which* fiscal decisions are considered (Alesina and Tabellini (2007) and Eggertsson and Borgne (2010) discuss what kinds of decisions are appropriate to delegate from politicians to unelected technocrats). Our model stresses the multidimensionality of fiscal policy, and envisions the delegation of a single, quantitative decision to the IFA: the choice of the current deficit. This

⁹Alesina and Tabellini (2007) show that this may reflect the career concerns of the technically-skilled bureaucrats who lead them.

¹⁰See the “Fiscal Compact” treaty, European Council (2012). In accord with the treaty, the Spanish government proposed legislation in June 2013 to establish a fiscal monitoring council; see Europa Press (28 June 2013).

¹¹Proposals in the first class include von Hagen and Harden (1995); Eichengreen, Hausmann, and von Hagen (1999); and Wyplosz (2005); those in the second class include Ball (1996); Gruen (1997); Seidman and Lewis (2002); Wren-Lewis (2002); and Costain and de Blas (2012a). Calmfors (2003) considers proposals of both types.

leaves the allocation of spending across different types of services (involving political and distributional choices) within the democratic process.

In contrast, many high-profile calls for European monetary mechanisms to prevent speculative attacks have assumed that this requires moves towards full political integration in Europe; see for example De Grauwe (2012), Soros (10 April 2013), and Pisani-Ferry (2012). We agree that fiscal reforms are essential to backstop an enhanced monetary policy, but we argue that the necessary reforms are much more limited than is commonly supposed. What is essential is that European authorities have the ability to ensure long-run national budget balance, and for this it suffices that they control at least *one* fiscal instrument of sufficient power in each member state. In accord with the principle of subsidiarity, the remaining fiscal decisions can remain at the national level. Sims (September 20, 2012) has likewise stressed that fiscal discipline requires delegating from national governments to Europe some instrument with a strong impact on each national budget, but that further fiscal integration is neither necessary nor likely to prove politically feasible. Similarly, some limited European tax powers form an essential backstop for the Schoemaker and Gros (2012) framework for banking union, but further fiscal integration is not required for their proposal.

2 The economic environment

The economic environment follows Beetsma and Bovenberg (1999, BB99) closely, but is generalized to an arbitrary horizon T . Their paper, like Rogoff (1985) and Barro and Gordon (1983), is built around a reduced-form model of the macroeconomy. Our paper does not aim to explain the imperfections in public institutions' decisions, such as excessive impatience or deficit bias, which have been discussed extensively in the political economy literature. Instead, our purpose is to build a model that incorporates these features in a parsimonious way, in order to study how equilibrium outcomes differ across games in which policy variables are controlled by different sets of institutions. Our stripped-down model permits a detailed, analytical comparison of perfect foresight equilibria across different policy regimes, to reveal how systematic policy biases are damped or enhanced by different institutional configurations.¹²

¹²Our decision to study systematic biases analytically in a reduced-form model implies that we do not address the stabilization of shocks. Addressing optimal stabilization raises important additional issues that are likely to require further microfoundations. These include the tradeoff between simple and complex institutional objectives (Persson and Tabellini (1993)), and incentives that affect the optimal speed of response to shocks, such as the impact of taxes on competitiveness in a monetary union (Leith and Wren-Lewis (2011)).

Time is discrete; we consider both finite- and infinite-horizon models. Several regions $j \in \{1, 2, \dots, J\}$ each benefit from local public spending, and face region-specific budget constraints. These regions might be considered nations, or subnational areas. Together, they form a monetary union, in which a single inflation rate applies.

2.1 Social welfare and budget constraints

Let time t private-sector output in country j be $x_{j,t}$. We distinguish actual private output from its target value $\tilde{x}_{j,t}$ (the bliss point). Our main assumption about the macroeconomy is that actual output rises if inflation π_t is higher than expected inflation, π_t^e , and that it falls with tax distortions $\tau_{j,t}$:

$$x_{j,t} = \nu(\pi_t - \pi_t^e - \tau_{j,t}). \quad (1)$$

Social welfare decreases quadratically as output, inflation, and government services $g_{j,t}$ deviate from their target values. The target level of inflation is assumed to be zero. The loss function for region j is¹³

$$L_{Sj} = \sum_{t=0}^T \beta_S^t \{ \alpha_{\pi S} \pi_t^2 + (x_{j,t} - \tilde{x}_{j,t})^2 + \alpha_{gS} (g_{j,t} - \tilde{g}_{j,t})^2 \}. \quad (2)$$

Here $\tilde{g}_{j,t}$ is the target level for government spending, and $\alpha_{\pi S} > 0$ and $\alpha_{gS} > 0$ are weights representing the relative importance of deviations of inflation and public services from their targets; without loss of generality the weight on output deviations is set equal to one. The discount factor for social welfare is $\beta_S \equiv \frac{1}{1+\rho_S} < 1$.

Since we are modeling a set of independent states that lack consensus for full political integration, we assume that policy is constrained by a budget constraint for each region. This takes the form

$$d_{j,t} = R d_{j,t-1} + p_{j,t}^g g_{j,t} - \tau_{j,t} - \kappa \pi_t, \quad (3)$$

where $d_{j,t}$ is the real debt of region j at the end of period t (which must be paid off at time $t+1$). Our assumption that seignorage revenues $\kappa \pi_t$ are linear in inflation, independent of the debt level, effectively means that government bonds are issued in real terms. If bonds were nominal, seignorage would also include a term proportional to the product of inflation and debt. We make this assumption about seignorage revenues for analytical convenience only; by optimizing a quadratic objective under

¹³Alesina and Tabellini (1987) derive an output relation of the form (1) from a more complete model. Also, Leith and Wren-Lewis (2011) derive a social welfare function of the form (2) from a standard New Keynesian framework in which government spending enters the utility function.

linear constraints we can solve the model explicitly. But allowing for nominal debt would only strengthen our results, since it would increase the gains from surprise inflation, and thereby reinforce both the inflation and deficit biases in our model.

Note also that the real interest rate r is assumed constant, since quadratic preferences imply certainty equivalence, so that $R \equiv 1 + r$ represents the inverse of the time-preference factor of the savers in the economy. Therefore, we impose the parameter restriction $\beta_S R = 1$. Furthermore, if T is finite, then there is a final constraint

$$d_{j,T} = 0 \quad (4)$$

which simply says that the economy ends at time T , and therefore markets are unwilling to make loans at T , since these loans will not be paid back. In an infinite-horizon context, debt must instead respect the following “no-Ponzi” condition:

$$\lim_{t \rightarrow \infty} \frac{d_{j,t}}{R^t} \leq 0, \quad (5)$$

which says that interest payments on debt are sufficient to make it worthwhile for the private sector (with discount rate R^{-1}) to hold the bonds.

Total public services in region j , $g_{j,t}$, are a constant-elasticity aggregate of a variety of differentiated services $g_{j,k,t}$:

$$g_{j,t} = \left(\int_0^1 \omega_{j,k,t} (g_{j,k,t})^{\frac{\eta-1}{\eta}} dk \right)^{\frac{\eta}{\eta-1}}. \quad (6)$$

where the $\omega_{j,k,t} > 0$ are weights on the different services k . Spending in region j is financed by taxes in that region, $\tau_{j,t}$, and by a share of seignorage revenues $\kappa\pi_t$. Total government spending is a sum over all public goods, $\int_0^1 g_{j,k,t} dk$. Spending is allocated to minimize the cost of the public aggregate provided:

$$p_{j,t}^g g_{j,t} \equiv \min_{\{g_{j,k,t}\}_{k=0}^1} \int_0^1 g_{j,k,t} dk \quad \text{s.t.} \quad \left(\int_0^1 \omega_{j,k,t} (g_{j,k,t})^{\frac{\eta-1}{\eta}} dk \right)^{\frac{\eta}{\eta-1}} \geq g_{j,t} \quad (7)$$

Equation (7) serves to define the price of government services, $p_{j,t}^g$. We assume that $\omega_{j,k,t}$ is independently and identically distributed for all j , k , and t .

We consider two possible scenarios for the public spending decision. On one hand, the policy maker that allocates public spending may know the distribution of $\omega_{j,k,t}$, but not observe its realization. Then it is optimal to allocate spending equally across all goods, so that

$$p_{j,t}^g = q^H \equiv E\omega_{j,k,t}^{\frac{\eta}{\eta-1}}.$$

At the opposite extreme, the policy maker may observe $w_{j,k,t}$ before choosing $g_{j,k,t}$. In this case, it is optimal to allocate more spending to the most-demanded services, according to the first-order condition

$$\frac{g_{j,k,t}}{g_{j,l,t}} = \left(\frac{\omega_{j,k,t}}{\omega_{j,l,t}} \right)^\eta$$

This more efficient allocation makes aggregate public services less expensive:

$$p_{j,t}^g = q^L < q^H.$$

2.2 An omniscient, committed, cooperative Pareto planner

Given these objectives and constraints, we next establish a welfare benchmark for our model. For relevance in the European context, we consider a Ramsey planner who maximizes social welfare taking market equilibrium conditions *and region-specific budget constraints* as given. Our planner does not represent any existing European institution, as it has unrealistic advantages in information and decision-making, but it represents a benchmark against which hypothetical institutions can be compared. For this purpose, we study an *omniscient, committed, cooperative Pareto planner*:

- *Omniscient*: the planner observes $\omega_{j,k,t}$ before choosing $g_{j,k,t}$. This makes aggregate public spending relatively inexpensive: $p_{j,t}^g = q^L$.
- *Committed*: the planner can credibly commit to choose the inflation rate it has previously announced. Therefore the inflation rate chosen by the planner is the rate expected by the public: $\pi_t = \pi_t^e$.
- *Cooperative*: the planner chooses the policy variables for all regions $j \in \{1, \dots, J\}$, and thus internalizes any externalities across borders.
- *Pareto*: the planner obeys a distinct budget constraint for each region, maximizing social welfare insofar as this does not require transfers across regions.

We write the planner's value function as $V_{P,t}$, the maximized value of $-L_{Sj}$, summed across all regions j . The value attainable depends on the debt levels of the regions, according to the following Bellman equation:

$$\begin{aligned} V_{P,t}(\{d_{j,t-1}\}_{j=1}^J) = & \max_{\pi_t, \{d_{j,t}, \tau_{j,t}\}_{j=1}^J} \frac{-1}{2} \left\{ \alpha_{\pi S} \pi_t^2 + \frac{1}{J} \sum_{j=1}^J \left[(\nu \tau_{j,t} + \tilde{x}_{j,t})^2 \right. \right. \\ & \left. \left. + \alpha_{gS} \left(\frac{d_{j,t} - R d_{j,t-1} + \tau_{j,t} + \kappa \pi_t}{q_L} - \tilde{g}_{j,t} \right)^2 \right] \right\} + \beta_S V_{P,t+1}(\{d_{j,t}\}_{j=1}^J). \end{aligned} \quad (8)$$

The first-order conditions for inflation and for region- j taxes are

$$\alpha_{\pi S} \pi_t + \frac{1}{J} \sum_{j=1}^J \frac{\kappa \alpha_{gS}}{q_L} (g_{j,t} - \tilde{g}_{j,t}) = 0, \quad (9)$$

$$\nu (\nu \tau_{j,t} + \tilde{x}_{j,t}) + \frac{\alpha_{gS}}{q_L} (g_{j,t} - \tilde{g}_{j,t}) = 0. \quad (10)$$

At time $T < \infty$, the planner must obey the terminal condition (4), or if $T = \infty$ then debt must satisfy (5). In all previous periods $t < T$, $d_{j,t}$ is chosen to satisfy

$$-\frac{1}{J} \frac{\alpha_{gS}}{q_L} (g_{j,t} - \tilde{g}_{j,t}) + \beta_S \frac{\partial V_{P,t+1}}{\partial d_{j,t}} (\{d_{k,t}\}_{k=1}^J) = 0. \quad (11)$$

We can then use an envelope condition:

$$\frac{\partial V_{P,t}}{\partial d_{j,t-1}} (\{d_{k,t-1}\}_{k=1}^J) = \frac{1}{J} \frac{R \alpha_{gS}}{q_L} (g_{j,t} - \tilde{g}_{j,t}), \quad (12)$$

to obtain an Euler equation for government spending in each region:

$$g_{j,t} - \tilde{g}_{j,t} = \beta_S R (g_{j,t+1} - \tilde{g}_{j,t+1}). \quad (13)$$

Finally, the planner's choices of taxes $\tau_{j,t}$ and debt $d_{j,t}$ determine current public services $g_{j,t}$ through the period budget constraint:

$$d_{j,t} = R d_{j,t-1} + q_L g_{j,t} - \tau_{j,t} - \kappa \pi_t. \quad (14)$$

Two *intratemporal* properties of the planner's solution are easily seen from the first-order conditions. For each period and region, there is a fixed linear relationship between output and public spending:

$$\nu \hat{x}_{j,t} = \frac{\alpha_{gS}}{q_L} \hat{g}_{j,t}, \quad (15)$$

where $\hat{x}_{j,t} = x_{j,t} - \tilde{x}_{j,t}$ and $\hat{g}_{j,t} = g_{j,t} - \tilde{g}_{j,t}$ and are the deviations of output and public spending from their bliss points. There is also a constant linear relation between inflation and *average* public spending:

$$\alpha_{\pi S} \pi_t = -\frac{\kappa \alpha_{gS}}{q_L} \bar{\hat{g}}_t, \quad (16)$$

where $\bar{\hat{g}}_t = J^{-1} \sum_j \hat{g}_{j,t}$. Thus, inflation is positive as long as government spending is below its bliss point, on average across countries.

Rewriting the Euler equation in terms of inflation, and plugging the within-period relations (15)-(16) into the constraint (14), we obtain a simple difference equation governing the aggregate dynamics of the planner's problem:

$$\begin{pmatrix} \bar{d}_t \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} R & -\tilde{\kappa}_P \\ 0 & \frac{1}{\beta_S R} \end{pmatrix} \begin{pmatrix} \bar{d}_{t-1} \\ \pi_t \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \bar{\tilde{z}}_t, \quad (17)$$

where $\bar{d}_t = J^{-1} \sum_j d_{j,t}$, $\bar{z}_t = J^{-1} \sum_j (\hat{x}_{j,t}/\nu + q_L \hat{g}_{j,t})$, and

$$\tilde{\kappa}_P = \kappa + \frac{\alpha_{\pi S}}{\kappa \alpha_{gS}} \left(q_L^2 + \frac{\alpha_{gS}}{\nu^2} \right) \quad (18)$$

If \bar{z} is constant, then the dynamics (17) have a steady state at zero inflation with

$$\bar{d}_{ss} = -\frac{\bar{z}}{r}. \quad (19)$$

At this steady state, assets are so high (debt is so negative) that the bliss points of output and public spending are affordable with interest earnings alone. The eigenvalues around this steady state are R and $(\beta_S R)^{-1}$. Assuming that the second eigenvalue is one, the system is saddle-path stable, and therefore has a unique equilibrium.

We can then summarize perfect foresight equilibrium behavior as follows.

- In an **infinite horizon** context, assuming $\beta_S R = 1$, \bar{d}_{SS} is *not* the only steady state. Instead, any debt level is a steady state (both in aggregate, and for each region j separately).¹⁴ Debt is held constant by setting a constant inflation rate $\pi = (\bar{z} + r\bar{d})/\tilde{\kappa}_P$, thus smoothing seignorage distortions over time. (Likewise, distortions in output and spending are smoothed over time in each country, at a level consistent with unchanging debt.)
- In a **finite horizon** context, assuming $\beta_S R = 1$, the planner smoothes seignorage distortions over time by setting a constant inflation rate π . Inflation is chosen so that aggregate debt hits zero at time T , $\bar{d}_T = 0$, given the aggregate constraint $\bar{d}_t = R\bar{d}_{t-1} - \tilde{\kappa}_P \pi + \bar{z}$. The required inflation rate is

$$\pi = \frac{1}{\tilde{\kappa}_P} \left(\bar{z} + \frac{R^{T+1}}{R^{T+1} - 1} r \bar{d}_{-1} \right).$$

(Likewise, distortions in output and spending are smoothed over time in each country, at a level consistent with $d_{j,T} = 0$ for all j .)

3 Policy games

3.1 Policy makers' objectives

Next, we consider equilibria in which several policy makers interact. Each one acts to minimize a loss function that resembles (2), but they may have different discount factors or different weighting coefficients on the loss terms.

¹⁴The existence of a continuum of steady states (or a random walk) for optimal debt is a standard finding; see for example Benigno and Woodford (2003).

First, there is a central bank C , which chooses inflation for the whole monetary union. The bank sums losses symmetrically across all J regions:

$$L_C = \sum_{t=0}^T \beta_C^t \left\{ J\alpha_{\pi C}\pi_t^2 + \sum_{j=1}^J [(x_{j,t} - \tilde{x}_{j,t})^2 + \alpha_{gC}(g_{j,t} - \tilde{g}_{j,t})^2] \right\}. \quad (20)$$

Second, each region j has a government G_j which chooses each type of public spending $g_{j,k,t}$, and thereby chooses aggregate public spending $g_{j,t}$. It may also be responsible for choosing the tax rate, depending on the institutional scenario considered. The government's loss function L_{G_j} only reflects terms involving region j :

$$L_{G_j} = \sum_{t=0}^T \beta_G^t \left\{ \alpha_{\pi G}\pi_t^2 + (x_{j,t} - \tilde{x}_{j,t})^2 + \alpha_{gG}(g_{j,t} - \tilde{g}_{j,t})^2 \right\}. \quad (21)$$

Third, we consider the possibility of a debt-averse fiscal authority. The fiscal authority may be established by and for region j , in which case we will call it F_j , or it may be a union-wide institution, in which case we will call it F . A regional authority F_j is assumed to care only about regional variables, having the loss function

$$L_{F_j} = \sum_{t=0}^T \beta_F^t \left\{ \alpha_{\pi F}\pi_t^2 + \alpha_{dF}(d_{j,t} - \tilde{d}_{j,t})^2 + (x_{j,t} - \tilde{x}_{j,t})^2 + \alpha_{gF}(g_{j,t} - \tilde{g}_{j,t})^2 \right\}. \quad (22)$$

This authority cares about the same terms as the society and government of region j , but it also cares about the region's real debt $d_{j,t}$, suffering a loss if debt deviates from its target value $\tilde{d}_{j,t}$. The level of the target plays little role in the analysis, so we simply set $\tilde{d}_{j,t} = 0$. A debt term in the fiscal authority's preferences is consistent with a mandate for long-run budget balance, since respecting an intertemporal budget constraint means that real debt cannot be too explosive. In contrast, the intertemporal budget constraint says little about the time path of the deficit; indeed, optimal taxation theory highlights the role of the deficit as a shock absorber, which may fluctuate strongly in order to allow other, distorting instruments to be smoothed.

Alternatively, if there is a single union-wide fiscal authority, we assume that it sums losses symmetrically across all regions:

$$L_F = \sum_{t=0}^T \beta_F^t \left\{ J\alpha_{\pi F}\pi_t^2 + \sum_{j=1}^J \left[\alpha_{dF}(d_{j,t} - \tilde{d}_{j,t})^2 + (x_{j,t} - \tilde{x}_{j,t})^2 + \alpha_{gF}(g_{j,t} - \tilde{g}_{j,t})^2 \right] \right\} \quad (23)$$

Note that this function includes a separate term for each region's debt, reflecting a concern for budget balance in each individual region, above and beyond its concern for union-wide budget balance. In other words, the fiscal authority is "Paretian", like the OCCPP planner.

Table 1: *Baseline parameter assumptions*

	Society and planner	Central bank	Government	Fiscal authority
Discount factor β_i	$0 < \beta_S < 1$ $\beta_S R = 1$	$\beta_C = \beta_S$	$0 < \beta_G < \beta_S$	$\beta_G < \beta_F \leq \beta_S$
Spending coefficient α_{gi}	$\alpha_{gS} > 0$	$\alpha_{gC} = \alpha_{gS}$	$\alpha_{gG} = \alpha_{gS}$	$\alpha_{gF} = \alpha_{gS}$
Inflation coefficient $\alpha_{\pi i}$	$\alpha_{\pi S} > 0$	$\alpha_{\pi C} > \alpha_{\pi S}$	$\alpha_{\pi G} = \alpha_{\pi S}$	$\alpha_{\pi F} = \alpha_{\pi S}$
Debt coefficient α_{di}	$\alpha_{dS} = 0$	$\alpha_{dC} = 0$	$\alpha_{dG} = 0$	$\alpha_{dF} > 0$
Coefficients of loss functions for agents $i \in \{S, C, G, F\}$				

3.1.1 Parameter assumptions

All these policy-making institutions are essentially benevolent, valuing the same goals as society and the planner. However, their different roles may imply some differences in priorities, reflected in the coefficients shown in Table 1. The central bank C and the fiscal authority F have the same degree of patience as society, but the government may be more impatient, due to the short time horizons of electoral politics. This is one reason why the government’s decisions may exhibit deficit bias. All three institutions $i \in \{C, G, F\}$ value public spending to the same degree that society does. But the central bank is assumed to have a mandate to achieve a target inflation rate; it therefore weighs losses from inflation variability more strongly than society does.¹⁵ Likewise, even though the debt level does not directly affect social welfare, we assume that the fiscal authority has a mandate to stabilize debt around some target level, and that this is reflected in its preferences. Therefore we include a positive coefficient on deviations of debt from target in its objective function.

By themselves, the baseline parameter assumptions stated in Table 1 do not guarantee the existence of an equilibrium with intuitively reasonable properties. This requires some additional conditions.

- We say that governments exhibit *moderate impatience* when the discount rate satisfies the following inequality:

$$\beta_G R^2 > 1. \tag{24}$$

We will assume throughout that (24) holds, because otherwise the government’s value function from (27) is unbounded in an infinite horizon context. In other

¹⁵Alesina and Tabellini (2007) discuss why society may prefer to delegate tasks with quantitative, verifiable objectives to bureaucrats, instead of leaving them up to the democratic government.

words, regardless of any other general equilibrium interactions, if (24) is not satisfied, then the government's partial equilibrium decision problem is not well defined for the $T = \infty$ case.

- We say that the central bank exhibits *moderate inflation aversion* when its preferences satisfy the following inequality:

$$\alpha_{\pi C} < \frac{1 + \kappa}{\kappa} \alpha_{\pi S}. \quad (25)$$

Throughout Sec. 3, we will assume that (25) holds, because it is central to the common pool problem of BB99, on which we base our model. As in Chari and Kehoe (2007), governments anticipate that the central bank will adjust inflation in response to their debt choices. If inflation rises *more than is optimal* when debt increases (which is what *moderate inflation aversion* implies) then a single government will hold down its debt in order to avoid the central bank's inefficient reaction; but as $J \rightarrow \infty$ this effect disappears, since each government regards its own debt as negligible. We build on this particular common pool problem for simplicity, and for consistency with BB99. But we will generalize to allow for another version of the common pool problem in Section 4.

3.2 The game of BB99

Beetsma and Bovenberg (1999) assume that in period t , the central bank chooses π_t , while the governments choose τ_{jt} and d_{jt} . Each government then spends the resources it has available, as given by the budget constraint (3). Market expectations are determined at the beginning of the period, rationally anticipating the outcome of the game between the bank and the governments.

Each policy maker's value is a function of the state of the economy, which includes the debt of each region j . We search for an equilibrium in which no other state variable is needed; that is, we rule out consideration of equilibria with more complex forms of history dependence, such as reputational effects. We call the central bank's value function $V_{C,t}$. Eliminating $x_{j,t}$ and $g_{j,t}$ using (1) and (3), its Bellman equation is:

$$V_{C,t}(\{d_{j,t-1}\}_{j=1}^J) = \max_{\pi_t} \frac{-1}{2} \left\{ \alpha_{\pi C} \pi_t^2 + \frac{1}{J} \sum_{j=1}^J \left[(\nu(\pi_t - \pi_t^e - \tau_{j,t}) - \tilde{x}_{j,t})^2 + \alpha_{gC} \left(\frac{d_{j,t} - R d_{j,t-1} + \tau_{j,t} + \kappa \pi_t}{q_L} - \tilde{g}_{j,t} \right)^2 \right] \right\} + \beta_C V_{C,t+1}(\{d_{j,t}\}_{j=1}^J) \quad (26)$$

Since policy makers cannot commit, this problem distinguishes between actual inflation π_t and expected inflation π_t^e . Government j 's value $V_{Gj,t}$ is governed by a similar Bellman equation, which determines taxes and debt for country j :

$$V_{Gj,t}(\{d_{k,t-1}\}_{k=1}^J) = \max_{\tau_{j,t}, d_{j,t}} \frac{-1}{2} \left\{ \alpha_{\pi G} \pi_t^2 + (\nu(\pi_t - \pi_t^e - \tau_{j,t}) - \tilde{x}_{j,t})^2 \right. \\ \left. + \alpha_{gG} \left(\frac{d_{j,t} - R d_{j,t-1} + \tau_{j,t} + \kappa \pi_t}{q_L} - \tilde{g}_{j,t} \right)^2 \right\} + \beta_G V_{Gj,t+1}(\{d_{k,t-1}\}_{k=1}^J) \quad (27)$$

The first-order condition for the central bank is

$$\alpha_{\pi C} \pi_t + \frac{1}{J} \sum_{j=1}^J \left[\nu(\nu(\pi_t - \pi_t^e - \tau_{j,t}) - \tilde{x}_{j,t}) + \frac{\kappa \alpha_{gC}}{q_L} (g_{j,t} - \tilde{g}_{j,t}) \right] = 0. \quad (28)$$

Compared with the social planner's necessary condition (9), we see an additional term relating to the central bank's incentive to set inflation unexpectedly high. Government j 's optimality condition for taxes is

$$-\nu(\nu(\pi_t - \pi_t^e - \tau_{j,t}) - \tilde{x}_{j,t}) + \frac{\alpha_{gG}}{q_L} (g_{j,t} - \tilde{g}_{j,t}) = 0. \quad (29)$$

Like the social planner, the government must obey the terminal budget constraint (4) or (5). For all periods $t < T$, government j sets

$$-\frac{\alpha_{gG}}{q_L} (g_{j,t} - \tilde{g}_{j,t}) + \beta_G \frac{\partial V_{Gj,t+1}}{\partial d_{j,t}}(\{d_{k,t}\}_{k=1}^J) = 0. \quad (30)$$

Its choices of taxes $\tau_{j,t}$ and debt $d_{j,t}$ then determine current spending $g_{j,t}$ according to the period budget constraint:

$$d_{j,t} = q_L g_{j,t} + R d_{j,t-1} - \tau_{j,t} - \kappa \pi_t. \quad (31)$$

The presence of multiple policy makers of non-negligible size implies extra terms in the value derivative in (30) which do not appear in the planner's derivative (12). To compute this derivative, we can ignore how $d_{j,t}$ impacts $\tau_{j,t+1}$ and $d_{j,t+1}$; these effects have zero marginal value, by the envelope theorem. But we cannot ignore the fact that changing $d_{j,t}$ will alter the central bank's choice of π_{t+1} , and other regions' debt choices $d_{k,t+1}$, for $k \neq j$; these interactions alter the marginal value of changing $d_{j,t}$. Differentiating (27), we obtain:

$$\frac{\partial V_{Gj,t}}{\partial d_{j,t-1}} = \frac{R \alpha_{gG}}{q_L} (g_{j,t} - \tilde{g}_{j,t}) - \left(\alpha_{\pi G} \pi_t + \frac{\kappa \alpha_{gG}}{q_L} (g_{j,t} - \tilde{g}_{j,t}) \right) \frac{\partial \pi_t}{\partial d_{j,t-1}} + \beta_G \sum_{k \neq j} \frac{\partial V_{Gj,t+1}}{\partial d_{k,t}} \frac{\partial d_{k,t}}{\partial d_{j,t-1}}. \quad (32)$$

Note that we do not need to track how π_t^e varies with $d_{j,t-1}$. Any change in π_t^e will cancel with a corresponding change in π_t , since under rational expectations $\pi_t^e = \pi_t$ for *any* value of $d_{j,t-1}$.¹⁶ Next, yet another derivative appears in (32): the marginal effect $\frac{\partial V_{Gj,t+1}}{\partial d_{k,t}}$ of region k 's debt on government j 's value. Differentiating (27) again yields:

$$\frac{\partial V_{Gj,t}}{\partial d_{k,t-1}} = - \left(\alpha_{\pi G} \pi_t + \frac{\kappa \alpha_{gG}}{q_L} (g_{j,t} - \tilde{g}_{j,t}) \right) \frac{\partial \pi_t}{\partial d_{k,t-1}} + \beta_G \sum_{l \neq j} \frac{\partial V_{Gj,t+1}}{\partial d_{l,t}} \frac{\partial d_{l,t}}{\partial d_{k,t-1}}. \quad (33)$$

Equations (28)-(29) are purely intratemporal, so they can be jointly simplified. Assuming $\alpha_{gG} = \alpha_{gS}$, the relation between output and spending is the same as in the social planner's problem:

$$\nu \hat{x}_{j,t} = \frac{\alpha_{gG}}{q_L} \hat{g}_{j,t}. \quad (34)$$

Plugging (34) into (28), we can then calculate inflation in terms of the average deviation of public spending from its bliss point:

$$\alpha_{\pi C} \pi_t = - \left(\frac{\alpha_{gG} + \kappa \alpha_{gC}}{q_L} \right) \bar{\hat{g}}_t \quad (35)$$

Comparing with the planner's first-order condition (16), the right-hand side of (35) contains an extra term reflecting the incentive for surprise inflation. On the other hand, a high inflation aversion coefficient on the left-hand side tends to restrain inflation. We can summarize the tradeoff between these two effects as follows:

- Under the baseline parameterization of Table 1, and assuming that the central bank displays *moderate inflation aversion*, the BB99 equilibrium exhibits a higher ratio of inflation to public spending distortions than the planner's solution does.

3.2.1 Solution when $J = \infty$

Using the fact that value functions should be quadratic, and policy functions linear, the model can be computed backwards for any J . But the solution is especially simple if $J = \infty$ or if $J = 1$. First, neither inflation nor other regions' debt will respond to region j 's debt if region j is negligibly small. So when $J = \infty$, (32) simplifies to

$$\frac{\partial V_{Gj,t}}{\partial d_{j,t-1}} = \frac{R \alpha_{gG}}{q_L} (g_{j,t} - \tilde{g}_{j,t}). \quad (36)$$

Then, using (30) and (35), inflation obeys a very simple Euler equation:

$$\pi_t = \beta_G R \pi_{t+1}. \quad (37)$$

¹⁶Recall that, as in BB99, π_t^e represents an expectation at the beginning of t , after time $t-1$ choices and time t shocks have been revealed.

Summing debt across regions, the matrix dynamics of the model are

$$\begin{pmatrix} \bar{d}_t \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} R & -\tilde{\kappa} \\ 0 & \frac{1}{\beta_G R} \end{pmatrix} \begin{pmatrix} \bar{d}_{t-1} \\ \pi_t \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \bar{z}_t, \quad (38)$$

where \bar{d}_t and \bar{z}_t were defined earlier, and

$$\tilde{\kappa} \equiv \kappa + \left(q_L^2 + \frac{\alpha_{gG}}{\nu^2} \right) \frac{\alpha_{\pi C}}{\alpha_{gG} + \kappa \alpha_{gC}}. \quad (39)$$

Notice:

- Assuming $\beta_G < \beta_S$, inflation grows faster in the BB99 economy than it does in the OCCPP planning solution.
- Under the baseline parameterization of Table 1, $\tilde{\kappa}^P > \tilde{\kappa}$ if and only if the central bank exhibits *moderate inflation aversion*.

Both of these observations point to overaccumulation of debt in the BB99 economy. First, faster inflation growth due to greater impatience means less inflationary financing in the short run than in the long run, consistent with a short-run debt build-up, to be paid off in the long run. Second, $\tilde{\kappa}^P > \tilde{\kappa}$ shows that for any current pair (d_{t-1}, π_t) , if $\pi_t > 0$, then the resulting debt level d_t is lower in the social planner's solution than in the BB99 economy. This second effect, as we emphasized above, relies on the assumption that the inflation aversion of the central bank is lower than optimal. As in Chari and Kehoe (2007), this aspect of deficit bias is a side-effect of inflation bias.

Now, assuming \bar{z} is constant, these dynamics have the same steady state as the social planner's problem does, with zero inflation and debt level $\bar{d}_{ss} = -\frac{\bar{z}}{r}$. The dynamics around the steady state are governed by the eigenvalues R and $(\beta_G R)^{-1}$, which are *both explosive*. Therefore,

- In an **infinite horizon** context, *no perfect foresight equilibrium with nonzero bondholdings exists*. With one predetermined variable (debt) and one jump variable (inflation), existence of a (unique) infinite-horizon equilibrium would require one stable and one unstable eigenvalue. Intuitively, the impatience of the government is inconsistent with the discount rate of the bondholders in this economy, so there is no infinite-horizon equilibrium in which the government's debt is held.
- In a **finite horizon** context, inflation explodes at a constant rate over time, as do output and spending distortions. That is, the impatient government chooses low distortions initially; to pay off the resulting stock of debt, over time it must increase inflation and taxes, while decreasing public services. The initial inflation rate is chosen so that aggregate debt hits zero at time T , $\bar{d}_T = 0$, given the aggregate dynamics $\bar{d}_t = R\bar{d}_{t-1} - \tilde{\kappa}\pi + \bar{z}$.

Focusing on constant \bar{z}_t is reasonable, since our paper seeks to analyze systematic deficit bias. When \bar{z}_t is constant, we can show that the initial buildup of debt is excessive, compared with the planner's solution (see the appendix for the proof):

Proposition 1. Suppose $T < \infty$, $J = \infty$, $\beta_G < \beta_S$, and $\bar{z}_t = \bar{z}$ is a constant. Then, starting from the same positive initial debt level $d_{-1} > 0$, debt will be strictly higher at all times $t \in \{0, 1, \dots, T - 1\}$ in the BB99 model than it is in the OCCPP planning solution.

3.2.2 Solution when $J = 1$

It is also relatively simple to analyze the case of a single region, $J = 1$. This case is informative, since by comparing $J = 1$ with $J = \infty$ while holding other factors fixed, we see the effects of joining a large monetary union. With only one region under consideration, the last term of equation (32) drops out, and so (33) is not needed. We can then derive the following Euler equation for public spending:

$$\frac{\alpha_{gG}}{q_L} \hat{g}_t = \beta_G R \frac{\alpha_{gG}}{q_L} \hat{g}_{t+1} - \beta_G \left(\alpha_{\pi G} \pi_{t+1} + \frac{\kappa \alpha_{gG}}{q_L} \hat{g}_{t+1} \right) \frac{\partial \pi_{t+1}}{\partial d_t}. \quad (40)$$

(The index j is suppressed here, since now there is only one region.) Using (35), we can restate (40) in terms of inflation only:

$$\pi_t = \beta_G \left(R + \gamma \frac{\partial \pi_{t+1}}{\partial d_t} \right) \pi_{t+1}. \quad (41)$$

where

$$\gamma = \kappa \left(\frac{\alpha_{\pi G}}{\alpha_{\pi C}} \left(\frac{\alpha_{gG} + \kappa \alpha_{gC}}{\kappa \alpha_{gG}} \right) - 1 \right). \quad (42)$$

We observe:

- Suppose the baseline parameter assumptions of Table 1 hold. Then $\gamma < 1$, and $\gamma > 0$ if and only if the central bank displays *moderate inflation aversion*.

The matrix dynamics of the model are summarized by a system very similar to those seen previously:

$$\begin{pmatrix} d_t \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} R & -\tilde{\kappa} \\ 0 & \left[\beta_G \left(R + \gamma \frac{\partial \pi_{t+1}}{\partial d_t} \right) \right]^{-1} \end{pmatrix} \begin{pmatrix} d_{t-1} \\ \pi_t \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \bar{z}_t. \quad (43)$$

Notice that *moderate inflation aversion* ($\gamma > 0$) will make inflation grow more slowly for $J = 1$ than for $J = \infty$ as long as the central bank increases inflation in response to greater debt ($\frac{\partial \pi_{t+1}}{\partial d_t} > 0$). Now, note that in this linear-quadratic model all control variables should be linear functions of the states; and in particular, in an infinite-horizon context $\frac{\partial \pi_{t+1}}{\partial d_t}$ will simply be a constant. Lemma 1 states that moderate inflation aversion suffices, but is not actually necessary, for this derivative to be positive.

Lemma 1. Suppose $-\frac{\kappa}{2} < \gamma$. Then in any finite- or infinite-horizon perfect foresight equilibrium of the BB99 game with $J = 1$, the response of inflation to debt $\frac{\partial \pi_{t+1}}{\partial d_t}$ is positive at all times.

Given these results, we can now conclude that under moderate inflation aversion, the model with a single region has lower inflation growth than the model with $J = \infty$. Since the $\tilde{\kappa}$ term is the same in the $J = 1$ and the $J = \infty$ cases, the high inflation growth of the $J = \infty$ case is associated with lower initial inflation, and higher final inflation, and therefore implies that debt initially accumulates, to be paid off later when inflation is high. Therefore:¹⁷

Proposition 2. Suppose $\beta_G < \beta_S$. Then:

- (a.) A perfect foresight equilibrium exists when $J = 1$ if $T < \infty$ or if γ is sufficiently large.
- (b.) Suppose $T < \infty$, and that the central bank exhibits *moderate inflation aversion*. Then, starting from the same positive per capita debt level $\bar{d}_{-1} > 0$, per capita debt will be strictly higher at all times $t \in \{0, 1, \dots, T - 1\}$ if $J = \infty$ than it would be if $J = 1$.

In other words, under moderate inflation aversion, a single region is less subject to deficit bias than a monetary union (large J). This difference is illustrated for a finite-horizon numerical example assuming “moderate impatience” and “moderate inflation aversion”¹⁸, comparing $J = 1$ with $J = 20$, in Figure 1. We see that inflation, government services, and output vary less over time in the $J = 1$ economy, and that debt declines roughly monotonically to zero by time T . In contrast, with $J = 20$, inflation rises steadily with time, as a debt burden accumulates and then must eventually be paid off; meanwhile government services and output decline steadily over time.

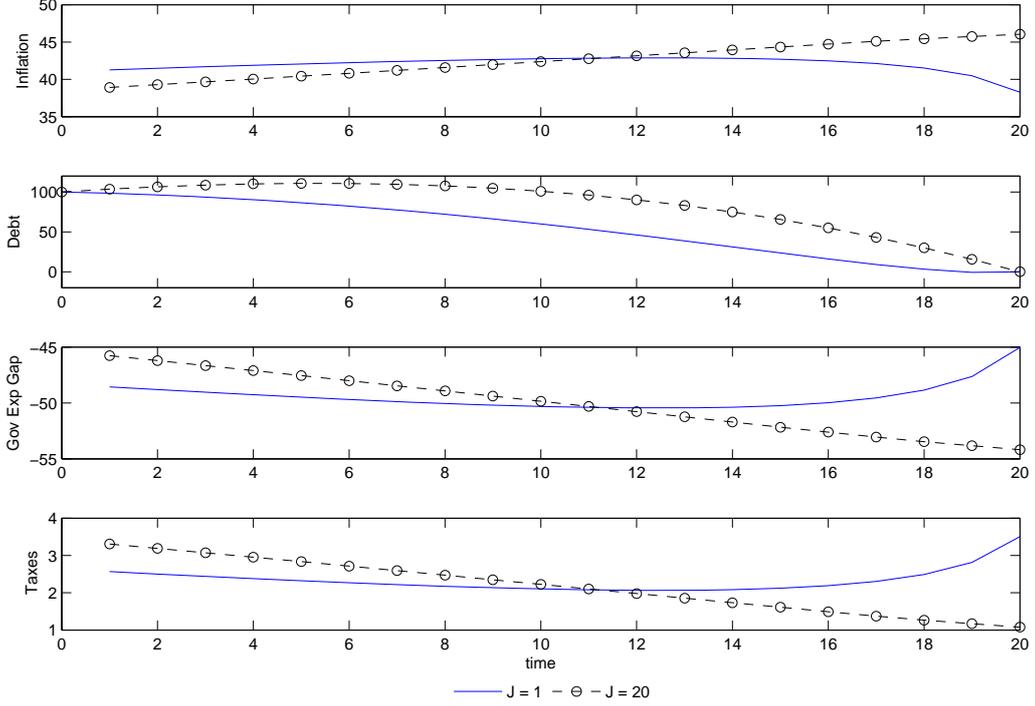
3.3 A game with a fiscal authority

Next, we solve a game in which debt is controlled by an independent debt-averse fiscal authority. The central bank’s Bellman equation is given by (26), as in the BB99 game. The Bellman equation of government j is identical to (27), except that now the only

¹⁷Result analogous to Prop. 2(b) are reported in the BB99 paper.

¹⁸The parameter values used in the numerical results are: $\beta_S = 0.96$, $R = 1/\beta_S$, $\nu = 1.94$, $q = 1$, $\beta_G = 0.99\beta_S$, $\alpha_{gC} = \alpha_{gG} = \alpha_{\pi G} = 1$, $\alpha_{\pi C} = 2 * \alpha_{\pi G}$, $\tilde{x} = -30$, $\tilde{g} = 100$, $d_0 = 100$ and $d_{20} = 0$.

Figure 1: *Example: effects of monetary union.*



Note. Dynamic simulation, 20 periods, comparing BB99 equilibrium for a single region ($J = 1$), with equilibrium for a monetary union with 20 members.

choice variable is τ_{jt} . A fiscal authority in region j would be described by the following Bellman equation:

$$V_{Fj,t}(\{d_{k,t-1}\}_{k=1}^J) = \max_{d_{j,t}} \frac{-1}{2} \left\{ \alpha_{\pi F} \pi_t^2 + \alpha_{dF} d_{j,t}^2 + (\nu(\pi_t - \pi_t^e - \tau_{j,t}) - \tilde{x}_{j,t})^2 \right. \\ \left. + \alpha_{gF} \left(\frac{d_{j,t} - R d_{j,t-1} + \tau_{j,t} + \kappa \pi_t}{q_L} - \tilde{g}_{j,t} \right)^2 \right\} + \beta_F V_{Fj,t+1}(\{d_{k,t}\}_{k=1}^J) \quad (44)$$

Alternatively, a fiscal authority for the union as a whole would be described by:

$$V_{F,t}(\{d_{j,t-1}\}_{j=1}^J) = \max_{\{d_{j,t}\}_{j=1}^J} \frac{-1}{2} \left\{ \alpha_{\pi F} \pi_t^2 + \frac{1}{J} \sum_{j=1}^J \left[\alpha_{dF} d_{j,t}^2 + (\nu(\pi_t - \pi_t^e - \tau_{j,t}) - \tilde{x}_{j,t})^2 \right. \right. \\ \left. \left. + \alpha_{gF} \left(\frac{d_{j,t} - R d_{j,t-1} + \tau_{j,t} + \kappa \pi_t}{q_L} - \tilde{g}_{j,t} \right)^2 \right] \right\} + \beta_F V_{F,t+1}(\{d_{j,t}\}_{j=1}^J) \quad (45)$$

As before, the first-order condition for the central bank is

$$\alpha_{\pi C} \pi_t + \frac{1}{J} \sum_{j=1}^J \left[\nu(\nu(\pi_t - \pi_t^e - \tau_{j,t}) - \tilde{x}_{j,t}) + \frac{\kappa \alpha_{gC}}{q_L} (g_{j,t} - \tilde{g}_{j,t}) \right] = 0, \quad (46)$$

and government j 's necessary condition for taxes is

$$-\nu(\nu(\pi_t - \pi_t^e - \tau_{j,t}) - \tilde{x}_{j,t}) + \frac{\alpha_{gG}}{q_L}(g_{j,t} - \tilde{g}_{j,t}) = 0. \quad (47)$$

Now suppose the fiscal authority is based in country j . Its terminal debt level must satisfy the "no Ponzi condition" (4) or (5) as in the previous models. In all periods $t < T$ it chooses debt to satisfy

$$-\frac{\alpha_{gF}}{q_L}(g_{j,t} - \tilde{g}_{j,t}) - \alpha_{dF}d_{j,t} + \beta_F \frac{\partial V_{Fj,t+1}}{\partial d_{j,t}}(\{d_{k,t}\}_{k=1}^J) = 0. \quad (48)$$

Compared with the model of BB99, we notice that (48) includes a new, debt-related term, derived from the fiscal authority's debt aversion. Finally, the government's tax decision $\tau_{j,t}$ and the fiscal authority's choice of debt $d_{j,t}$ jointly determine current spending $g_{j,t}$, via the period budget constraint:

$$d_{j,t} = q_L g_{j,t} + R d_{j,t-1} - \tau_{j,t} - \kappa \pi_t. \quad (49)$$

Since the players are not infinitesimal, each takes into account how its moves affect future moves by other players. Thus, the fiscal authority's marginal value of debt includes its impact on future inflation and taxes, and on other regions' future debts:

$$\begin{aligned} \frac{\partial V_{Fj,t}}{\partial d_{j,t-1}} &= \frac{R\alpha_{gF}}{q_L}(g_{j,t} - \tilde{g}_{j,t}) + \left[\nu(\nu(\pi_t - \pi_t^e - \tau_{j,t}) - \tilde{x}_{j,t}) - \frac{\alpha_{gF}}{q_L}(g_{j,t} - \tilde{g}_{j,t}) \right] \frac{\partial \tau_{j,t}}{\partial d_{j,t-1}} \\ &\quad - \left(\alpha_{\pi F} \pi_t + \frac{\kappa \alpha_{gF}}{q_L}(g_{j,t} - \tilde{g}_{j,t}) \right) \frac{\partial \pi_t}{\partial d_{j,t-1}} + \beta_F \sum_{k \neq j} \frac{\partial V_{Fj,t+1}}{\partial d_{k,t}} \frac{\partial d_{k,t}}{\partial d_{j,t-1}} \\ &= \frac{R\alpha_{gF}}{q_L} \hat{g}_{j,t} - \left(\alpha_{\pi F} \pi_t + \frac{\kappa \alpha_{gF}}{q_L} \hat{g}_{j,t} \right) \frac{\partial \pi_t}{\partial d_{j,t-1}} + \beta_F \sum_{k \neq j} \frac{\partial V_{Fj,t+1}}{\partial d_{k,t}} \frac{\partial d_{k,t}}{\partial d_{j,t-1}} \end{aligned} \quad (50)$$

The terms proportional to $\frac{\partial \tau}{\partial d}$ cancel out of (50) using the baseline parameter assumptions from Table 1. Specifically, assuming $\alpha_{gG} = \alpha_{gF}$, they match the terms in the government's first-order condition (47), and therefore sum to zero. Evaluating (50) also requires expressions for some cross derivatives, representing the marginal value to F_j of region k 's debt, for $k \neq j$. Again, the derivatives simplify if $\alpha_{gG} = \alpha_{gF}$:

$$\begin{aligned} \frac{\partial V_{Fj,t}}{\partial d_{k,t-1}} &= \left[\nu(\nu(\pi_t - \pi_t^e - \tau_{j,t}) - \tilde{x}_{j,t}) - \frac{\alpha_{gF}}{q_L}(g_{j,t} - \tilde{g}_{j,t}) \right] \frac{\partial \tau_{j,t}}{\partial d_{k,t-1}} \\ &\quad - \left(\alpha_{\pi F} \pi_t + \frac{\kappa \alpha_{gF}}{q_L}(g_{j,t} - \tilde{g}_{j,t}) \right) \frac{\partial \pi_t}{\partial d_{k,t-1}} + \beta_F \sum_{l \neq j} \frac{\partial V_{Fj,t+1}}{\partial d_{l,t}} \frac{\partial d_{l,t}}{\partial d_{k,t-1}} \\ &= - \left(\alpha_{\pi F} \pi_t + \frac{\kappa \alpha_{gF}}{q_L} \hat{g}_{j,t} \right) \frac{\partial \pi_t}{\partial d_{k,t-1}} + \beta_F \sum_{l \neq j} \frac{\partial V_{Fj,t+1}}{\partial d_{l,t}} \frac{\partial d_{l,t}}{\partial d_{k,t-1}}. \end{aligned} \quad (51)$$

If instead the fiscal authority is a union-wide institution, for $t < T$ it sets:

$$-\frac{1}{J} \frac{\alpha_{gF}}{q_L} (g_{j,t} - \tilde{g}_{j,t}) - \frac{1}{J} \alpha_{dF} d_{j,t} + \beta_F \frac{\partial V_{F,t+1}}{\partial d_{j,t}} (\{d_{k,t}\}_{k=1}^J) = 0. \quad (52)$$

Using (45), the marginal value of country j 's debt is given by

$$\begin{aligned} \frac{\partial V_{F,t}}{\partial d_{j,t-1}} &= \frac{R\alpha_{gF}}{q_L} \frac{1}{J} (g_{j,t} - \tilde{g}_{j,t}) + \frac{1}{J} \sum_{k=1}^J \left[\nu(\pi_t - \pi_t^e - \tau_{k,t}) - \tilde{x}_{k,t} - \frac{\alpha_{gF}}{q_L} (g_{k,t} - \tilde{g}_{j,t}) \right] \frac{\partial \tau_{j,t}}{\partial d_{j,t-1}} \\ &\quad - \frac{1}{J} \sum_{k=1}^J \left(\alpha_{\pi F} \pi_t + \frac{\kappa \alpha_{gF}}{q_L} (g_{j,t} - \tilde{g}_{j,t}) \right) \frac{\partial \pi_t}{\partial d_{j,t-1}} \\ &= \frac{R\alpha_{gF}}{Jq_L} \hat{g}_{j,t} - \frac{1}{J} \sum_{k=1}^J \left(\alpha_{\pi F} \pi_t + \frac{\kappa \alpha_{gF}}{q_L} \hat{g}_{j,t} \right) \frac{\partial \pi_t}{\partial d_{j,t-1}}. \end{aligned} \quad (53)$$

Here again, we have used $\alpha_{gG} = \alpha_{gF}$ and (47) to eliminate the $\frac{\partial \tau}{\partial d}$ terms. Notice also that no value function derivatives occur on the right-hand side of (53); they are eliminated using the envelope theorem.

The intratemporal first-order conditions can be simplified in the same way as in the BB99 model. Equation (47) becomes

$$\nu \hat{x}_{j,t} = \frac{\alpha_{gG}}{q_L} \hat{g}_{j,t} \quad (54)$$

which is identical to (34). Likewise, plugging (54) into (46), we obtain:

$$\alpha_{\pi C} \pi_t = - \left(\frac{\alpha_{gG} + \kappa \alpha_{gC}}{q_L} \right) \bar{\hat{g}}_t \quad (55)$$

which is identical to (35).

3.3.1 Solving the FA_j game when $J = \infty$

As in the BB99 model, calculating the value function derivatives generally requires a numerical solution. But the main results can be seen by considering the limiting case $J = \infty$, in which each region is infinitesimal, so neither inflation nor other regions' debt will respond to region j 's debt. Then the envelope condition (50) simplifies to

$$\frac{\partial V_{Fj,t}}{\partial d_{j,t-1}} = \frac{R\alpha_{gF}}{q_L} (g_{j,t} - \tilde{g}_{j,t}). \quad (56)$$

Plugging this expression into (48), we obtain an Euler equation for public spending:

$$\frac{\alpha_{gF}}{q_L} \hat{g}_{j,t} + \alpha_{dF} d_{j,t} = \beta_F R \frac{\alpha_{gF}}{q_L} \hat{g}_{j,t+1} \quad (57)$$

Averaging across regions, we can write (63) in terms of inflation and average debt:

$$\pi_t = \alpha_{dF} \left(\frac{\alpha_{gG} + \kappa\alpha_{gC}}{\alpha_{gF}\alpha_{\pi C}} \right) \bar{d}_t + \beta_F R \pi_{t+1}. \quad (58)$$

Comparing (37) with (58), we see that the fiscal authority has two inflation-inhibiting effects. First, at $d_{j,t} = 0$, inflation grows more slowly in the presence of the fiscal authority if $\beta_G < \beta_F$, that is, if the government is less patient than the fiscal authority. Second, for any $d_{j,t} > 0$ inflation grows more slowly in the presence of the fiscal authority as long as $\alpha_{dF} > 0$, that is, if the fiscal authority dislikes debt.

The dynamics can be summarized in matrix form as

$$\begin{pmatrix} \bar{d}_t \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} R & -\tilde{\kappa} \\ \frac{-\tilde{\alpha}}{\beta_F} & \frac{1+\tilde{\alpha}\tilde{\kappa}}{\beta_F R} \end{pmatrix} \begin{pmatrix} \bar{d}_{t-1} \\ \pi_t \end{pmatrix} + \begin{pmatrix} 1 \\ -\frac{\tilde{\alpha}}{\beta_F R} \end{pmatrix} \bar{z}_t, \quad (59)$$

where

$$\tilde{\alpha} \equiv \frac{\alpha_{dF}}{\alpha_{gF}} \left(\frac{\alpha_{gG} + \kappa\alpha_{gC}}{\alpha_{\pi C}} \right). \quad (60)$$

We now summarize some key observations about these dynamics.

Prop. 3. Suppose the baseline parameter assumptions hold, and $J = \infty$.

- (a.) The model with a fiscal authority *at the country level* behaves exactly as the BB99 model if $\beta_F = \beta_G$ and $\alpha_{dF} = 0$.
- (b.) If $T = \infty$ and $\beta_F R < 1$, then no perfect foresight equilibrium exists if $\alpha_{dF} = 0$. A unique perfect foresight equilibrium exists if α_{dF} is sufficiently large.
- (c.) Suppose $T < \infty$, $\beta_F > \beta_G$, $\alpha_{dF} > 0$, and $\bar{z}_t = \bar{z}$ is a constant. Then, starting from the same positive initial debt level $d_{-1} > 0$, debt will be strictly lower at all times $t \in \{0, 1, \dots, T-1\}$ in the regime with national fiscal authorities than it would be in the BB99 model.

Prop. 3(a) points out that separating the choice of taxation and deficits from the choice of spending has no effect if a *local* institution choosing taxes and deficits has preferences identical to the government (because if so, the set of first-order conditions defining equilibrium is equivalent).

To prove Prop. 3(b), we first show that both eigenvalues of the system (59) are real. We then show that both eigenvalues are greater than one if $\alpha_{dF} = 0$, but that one of the eigenvalues is less than one when α_{dF} is sufficiently large. That is, sufficiently strong debt aversion by the fiscal authority stabilizes debt dynamics, implying the existence of a unique equilibrium. The condition for “sufficiently large” is:

$$\alpha_{dF} > \frac{1}{\tilde{\kappa}} \left(\frac{\alpha_{\pi C} \alpha_{gF}}{\alpha_{gG} + \kappa\alpha_{gC}} \right) [R + \beta_F R - \beta_F R^2 - 1]. \quad (61)$$

Condition (61) reduces to $\alpha_{dF} > 0$ if we impose $\beta_F R = 1$, as in Table 1.

Part (c) is proved by relying again on the fact that the equilibrium policy functions should be linear in debt. For $T < \infty$, the coefficients of the linear function will change over time, but they can be calculated by working backwards from the last period T . In the appendix, we show that if the fiscal authority is less impatient than the government, and/or exhibits debt aversion, then (starting from a positive debt level) debt always remains higher in the BB99 economy than it does in the FA_j economy.

3.3.2 Solving the FA game

Next, we solve the model with a single fiscal authority at the level of the monetary union. To solve this case, note that given the linear form of the equilibrium, the response of inflation to any given country's debt must be $1/J$ times its response to aggregate debt.¹⁹ Thus the envelope condition (53) becomes

$$\frac{\partial V_{F,t}}{\partial d_{j,t-1}} = \frac{1}{J} \frac{R\alpha_{gF}}{q_L} \hat{g}_{j,t} - \left(\alpha_{\pi F} \pi_t + \frac{\kappa\alpha_{gF}}{q_L} \bar{g}_t \right) \frac{1}{J} \frac{\partial \pi_t}{\partial \bar{d}_{t-1}}, \quad (62)$$

so that the factor $1/J$ cancels out of the Euler equation:

$$\frac{\alpha_{gF}}{q_L} \hat{g}_{j,t} + \alpha_{dF} d_{j,t} = \beta_F R \frac{\alpha_{gF}}{q_L} \hat{g}_{j,t+1} - \beta_F \left(\alpha_{\pi F} \pi_{t+1} + \frac{\kappa\alpha_{gF}}{q_L} \bar{g}_{t+1} \right) \frac{\partial \pi_{t+1}}{\partial \bar{d}_t}. \quad (63)$$

We observe

- When debt is controlled by a single fiscal authority at the union level, the dynamics of the model are independent of the number of regions J .

If we now average over all regions, and substitute π for \bar{g} , we obtain the Euler equation in terms of inflation:

$$\pi_t = \beta_F \left(R + \gamma_F \frac{\partial \pi_{t+1}}{\partial \bar{d}_t} \right) \pi_{t+1} + \tilde{\alpha} \bar{d}_t, \quad (64)$$

where $\tilde{\alpha}$ was defined in (60), and

$$\gamma_F = \kappa \left(\frac{\alpha_{\pi F}}{\alpha_{\pi C}} \left(\frac{\alpha_{gG} + \kappa\alpha_{gC}}{\kappa\alpha_{gF}} \right) - 1 \right). \quad (65)$$

We observe:

- Suppose the baseline parameter assumptions of Table 1 hold. Then $\gamma_F = \gamma < 1$, and $\gamma_F > 0$ if and only if the central bank displays *moderate inflation aversion*.

¹⁹This simplification would no longer be valid if we allowed for other parameter differences across regions, other than their initial debt level.

Writing the dynamics in matrix form, we have

$$\begin{pmatrix} \bar{d}_t \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} R & -\tilde{\kappa} \\ \frac{-R\tilde{\alpha}}{\beta_F(R+\gamma_F\frac{\partial\pi_{t+1}}{\partial d_t})} & \frac{1+\tilde{\alpha}\tilde{\kappa}}{\beta_F(R+\gamma_F\frac{\partial\pi_{t+1}}{\partial d_t})} \end{pmatrix} \begin{pmatrix} \bar{d}_{t-1} \\ \pi_t \end{pmatrix} + \begin{pmatrix} 1 \\ -\frac{\tilde{\alpha}}{\beta_F(R+\gamma_F\frac{\partial\pi_{t+1}}{\partial d_t})} \end{pmatrix} \bar{\tilde{z}}_t. \quad (66)$$

This system combines two properties we have seen before. Like a model with fiscal authorities at the regional level, debt alters the dynamics of inflation, as long as the fiscal authority is debt averse ($\tilde{\alpha} > 0$). But in addition, inflation growth is affected by the term $\gamma_F\frac{\partial\pi_{t+1}}{\partial d_t}$, as in the BB99 model of a single region. This term is positive, slowing down inflation growth, if the central bank exhibits moderate inflation aversion and chooses $\frac{\partial\pi_{t+1}}{\partial d_t} > 0$. Lemma 2 shows that inflation responds positively to debt in the FA model under the same condition that applied to the BB99, $J = 1$ model.

Lemma 2. Suppose $-\frac{\kappa}{2} < \gamma$. Then in any finite- or infinite-horizon perfect foresight equilibrium of the FA game, the response of inflation to debt $\frac{\partial\pi_{t+1}}{\partial d_t}$ is positive at all times.

Inflation growth is also slowed down, relative to the BB99 baseline model, by the higher discount factor $\beta_F > \beta_G$. We summarize some key observations about system (66) as follows.

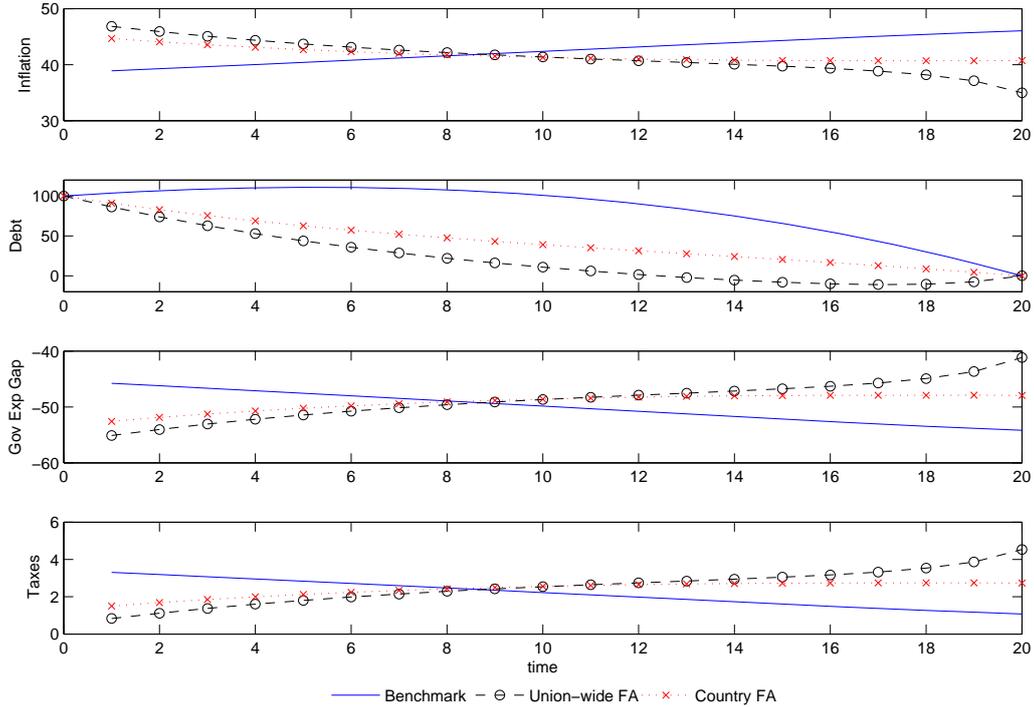
Proposition 4. Suppose the baseline parameter assumptions hold.

- (a.) The model with a single fiscal authority *at the union level* behaves exactly as the BB99 model with $J = 1$ if $\beta_F = \beta_G$ and $\alpha_{dF} = 0$.
- (b.) If $T = \infty$ and $\beta_F R \leq 1$, a unique perfect foresight equilibrium exists if α_{dF} is sufficiently large and/or if γ is sufficiently large.
- (c.) Suppose $T < \infty$, $\gamma_F > 0$, and $\bar{\tilde{z}}_t = \bar{\tilde{z}}$ is a constant. Then, fixing β_F and α_{dF} and starting from the same positive initial debt level $d_{-1} > 0$, debt will be strictly lower at all times $t \in \{0, 1, \dots, T-1\}$ under a union-wide fiscal authority than it would be with national fiscal authorities.

Figure 2 shows a finite-horizon numerical example comparing the baseline BB99 scenario (with $J = 20$, as in Figure 1) and scenarios with regional or centralized fiscal authorities assuming “moderate impatience” and “moderate inflation aversion”²⁰.

²⁰The parameter values used here are the same ones used in Figure 1. Additionally we set $\alpha_{dF} = 0.1$ and $\beta_F = 0.995\beta_S$.

Figure 2: *Example: effects of fiscal authority.*



Note. Dynamic simulation, 20 periods, comparing three policy configurations for a monetary union with 20 members: baseline BB99 equilibrium, equilibrium with fiscal authorities in each region, and equilibrium with a single central fiscal authority.

3.4 A game with a federal government

Next, we briefly compare the behavior of an independent fiscal authority solely concerned with budget balance with the behavior of a European federal government that controls all aspects of fiscal policy. Given our specification of the previous games, it now makes sense to assume that the budget constraint is

$$d_{j,t} = Rd_{j,t-1} + q_H g_{j,t} - \tau_{j,t} - s_{j,t} - \kappa \pi_t. \quad (67)$$

Government services now cost $q_H > q_L$, because taking all fiscal decisions at the union level implies a loss of local knowledge about spending needs. The budget constraint also reflects the possibility of transfers $s_{j,t}$ across regions. We could assume that the government remains “Paretian”, so that regions remain fully responsible for their own budgets, setting

$$s_{j,t} = 0 \quad (68)$$

for all j . But we could also consider a full “fiscal transfer union”, in which transfers are constrained only by

$$\sum_{j=1}^J s_{j,t} = 0. \quad (69)$$

The central bank’s Bellman equation now becomes:

$$\begin{aligned} V_{C,t}(\{d_{j,t-1}\}_{j=1}^J) = & \max_{\pi_t} \frac{-1}{2} \left\{ \alpha_{\pi C} \pi_t^2 + \frac{1}{J} \sum_{j=1}^J \left[(\nu(\pi_t - \pi_t^e - \tau_{j,t}) - \tilde{x}_{j,t})^2 \right. \right. \\ & \left. \left. + \alpha_{gC} \left(\frac{d_{j,t} - R d_{j,t-1} + \tau_{j,t} + s_{j,t} + \kappa \pi_t}{q_H} - \tilde{g}_{j,t} \right)^2 \right] \right\} + \beta_C V_{C,t+1}(\{d_{j,t}\}_{j=1}^J) \end{aligned} \quad (70)$$

The government solves

$$\begin{aligned} V_{G,t}(\{d_{j,t-1}\}_{j=1}^J) = & \max_{\{d_{j,t}, \tau_{j,t}, s_{j,t}\}_{j=1}^J} \frac{-1}{2} \left\{ \alpha_{\pi G} \pi_t^2 + \frac{1}{J} \sum_{j=1}^J \left[(\nu(\pi_t - \pi_t^e - \tau_{j,t}) - \tilde{x}_{j,t})^2 \right. \right. \\ & \left. \left. + \alpha_{gG} \left(\frac{d_{j,t} - R d_{j,t-1} + \tau_{j,t} + s_{j,t} + \kappa \pi_t}{q_H} - \tilde{g}_{j,t} \right)^2 \right] \right\} + \beta_G V_{G,t+1}(\{d_{j,t}\}_{j=1}^J) \end{aligned} \quad (71)$$

subject either to (68) or (69). Note that (71) is based on the assumption that the European federal government is democratic. Consistent with our earlier assumptions, we suppose that electoral politics tends to make the government more impatient than society as a whole. And since the government must choose between many competing uses of funds, there is no reason for it to be biased against debt, any more than an individual region’s government would be.

Given our previous results, it is easy to see how this setup will behave. If the government is Paretian, in the aggregate it acts just like our previous model of a single government (with composite preferences) interacting with a single central bank. Thus the dynamics are analogous to (43), except that they now refer to average debt over the whole union:

$$\begin{pmatrix} \bar{d}_t \\ \pi_{t+1} \end{pmatrix} = \begin{pmatrix} R & -\tilde{\kappa}_G \\ 0 & \left[\beta_G \left(R + \gamma \frac{\partial \pi_{t+1}}{\partial \bar{d}_t} \right) \right]^{-1} \end{pmatrix} \begin{pmatrix} \bar{d}_{t-1} \\ \pi_t \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tilde{z}_t^G. \quad (72)$$

where

$$\tilde{\kappa}_G \equiv \kappa + \left(q_H^2 + \frac{\alpha_{gG}}{\nu^2} \right) \frac{\alpha_{\pi C}}{\alpha_{gG} + \kappa \alpha_{gC}}, \quad (73)$$

$$\tilde{z}_t^G \equiv q_H \tilde{g}_t + \frac{\tilde{x}_t}{\nu}. \quad (74)$$

Note that these dynamics are independent of the number of regions, because the federal government acts as a single decision-maker that internalizes the common pool problem across regions. This is why the term $\gamma \frac{\partial \pi_{t+1}}{\partial d_t}$ appears in the inflation dynamics, as it does in the BB99 model with $J = 1$, slowing down the explosion of inflation by counteracting the impatience of the government.

A major disadvantage of this setup is the loss of “subsidiarity”: spending decisions are taken at the union level, where less information is available, and therefore public services are more expensive than they would be if they were allocated locally. The relation between inflation, public spending, and output would be

$$\bar{g}_t = - \left(\frac{\alpha_{\pi C} q_H}{\alpha_{gG} + \kappa \alpha_{gC}} \right) \pi_t, \quad (75)$$

$$\hat{x}_{j,t} = - \frac{\alpha_{gG}}{\nu} \left(\frac{\alpha_{\pi C}}{\alpha_{gG} + \kappa \alpha_{gC}} \right) \pi_t \quad (76)$$

Comparing with the corresponding relations for the BB99 economy, (34)-(35), which also apply in the economy with a fiscal authority, (75) and (76) show that the relation between inflation and output is unchanged, but that for any given level of inflation, the distance of government services from their bliss point is increased.

We can summarize these observations as follows.

- Like a union-wide fiscal authority, a federal government for the monetary union would internalize the common pool problem across regions. On the other hand, the federal government would tend to accumulate more debt insofar as democratic politics makes it more impatient and less debt averse than the fiscal authority.
- For a given level of inflation, the federal government would achieve the same level of output, but a lower level of government services, compared with the fiscal authority (and with the BB99 economy).

Intuitively, three forces restrain debt in the union-wide fiscal authority case: increased patience, debt aversion and elimination of the common-pool problem. In the federal government case, only the last of these three mechanisms applies. But beyond the effects on the debt level, the federal government also causes a decrease in the efficiency of public spending, insofar as less information is available at the centralized level for correctly allocating spending decisions.

3.4.1 Fiscal transfer union

If we instead consider a “fiscal transfer union”, in which nonzero transfers across regions occur, the aggregate dynamics are still given by (72). The only difference is at the regional level, at which all differences in distortions are eliminated. That is, $\hat{g}_{j,t}$ and $\hat{x}_{j,t}$ are equalized across all regions, so that the marginal utilities of output and public spending do not differ with j . This raises the level of social welfare in the aggregate, but it is not a Pareto improvement: regions with higher debt (or less favorable shocks $\tilde{g}_{j,t}$ or $\tilde{x}_{j,t}$) benefit at the expense of regions with less debt or more favorable shocks, from whom they receive transfers.²¹

4 Contagion and the common pool problem

Up to this point, our analysis has been based on a particular form of the budgetary common pool problem, taken from BB99 and Chari and Kehoe (2007), in which deficit bias arises only as a side-effect of inflation bias. Conveniently, this resulted in a linear-quadratic model that we were able to characterize analytically in some detail. But assuming that the central bank is insufficiently inflation averse does not seem like a realistic description of the institutional setup in Europe. Therefore, in this section we introduce a new form of the budgetary common pool problem, based on a more appealing mechanism: contagion in sovereign bond markets. Realistically, we now assume that the interest rate on sovereign bonds varies with the debt level. Outside of a monetary union, we suppose that each country’s sovereign interest rate depends only on its own debt. Inside a monetary union, we assume contagion increases: each member state’s interest rate depends on its own debt, but also on the debt levels of the other members. We model this contagion in the simplest and starkest way by treating the interest rate throughout the monetary union as a function only of the average debt level in the union.

This type of interest rate contagion implies a common pool problem in the monetary union. A country with its own monetary policy would try to avoid debt accumulation, in order to avoid an increase in the interest rate it pays. But a member of a large monetary union knows that its debt is only an infinitesimal part of the total, and therefore has no incentive to restrain its own debt accumulation even though a rise in the union’s total debt raises interest rates for all. Under this assumption, “moderate inflation aversion” of the central bank is no longer a necessary condition for inefficient

²¹Since the transfers $s_{j,t}$ are lump sum, only the present discounted value of transfers is determined by the model, not the actual time path of transfers.

excess debt in the monetary union. An independent fiscal authority's restraining effect on debt is therefore additionally beneficial in this context.

When we build these effects into our model, the optimization problems of the planner, the central bank, the governments, and the fiscal authority (or authorities) are identical to those considered in Section 3, except that now the interest rate varies with debt, instead of being a constant $r \equiv R - 1$. For a country j with an independent monetary policy, the interest rate on time t debt $d_{j,t}$ is

$$r_{j,t} \equiv R(d_{j,t}) - 1, \quad (77)$$

where R is an increasing function. In a monetary union, the interest rate on all countries' debt is instead

$$r_t \equiv R\left(J^{-1} \sum_{j=1}^J d_{j,t}\right) - 1. \quad (78)$$

We will impose one weak restriction on debt and interest rates in equilibrium. We say that the model exhibits *extreme dynamic inefficiency* if, at the steady-state debt level d_{ss} ,

$$R(d_{ss}) + R'(d_{ss})d_{ss} \leq 1. \quad (79)$$

This condition cannot hold unless d is sufficiently low, so that $d < 0$ and/or $R < 1$; if it holds, this means assets are so large that *saving less in steady state* would imply *more interest income in steady state*. Since we are focusing our study on economies with excessive debt, the extreme excess saving implied by (79) is of little interest. Therefore we simply rule out any parameterization that implies (79).

The first order conditions are analogous to those seen previously. The equation systems for the planner, for the individual country case (BB99, $J = 1$), for the monetary union without a fiscal authority (BB99, $J = \infty$), for the monetary union with country-level fiscal authorities (FA_j), and for the monetary union with a union-wide fiscal authority (FA), are as follows.

Social Planner - OCCPP

$$\begin{cases} \bar{d}_t = R(\bar{d}_{t-1})\bar{d}_{t-1} - \tilde{\kappa}_P\pi_t + \tilde{z} \\ \pi_t = \beta_S (R(\bar{d}_t) + R'\bar{d}_t) \pi_{t+1}. \end{cases} \quad (80)$$

BB99 - $J = 1$

$$\begin{cases} \bar{d}_t = R(\bar{d}_{t-1})\bar{d}_{t-1} - \tilde{\kappa}\pi_t + \tilde{z} \\ \pi_t = \beta_G \left(R(\bar{d}_t) + R'\bar{d}_t + \gamma \frac{\partial \pi_{t+1}}{\partial \bar{d}_t} \right) \pi_{t+1}. \end{cases} \quad (81)$$

BB99 - $J = \infty$

$$\begin{cases} \bar{d}_t = R(\bar{d}_{t-1})\bar{d}_{t-1} - \tilde{\kappa}\pi_t + \tilde{z} \\ \pi_t = \beta_G R(\bar{d}_t) \pi_{t+1}. \end{cases} \quad (82)$$

$FA_j - J = \infty$

$$\begin{cases} \bar{d}_t = R(\bar{d}_{t-1})\bar{d}_{t-1} - \tilde{\kappa}\pi_t + \tilde{z} \\ \pi_t = \tilde{\alpha}d_t + \beta_F R(\bar{d}_t)\pi_{t+1}. \end{cases} \quad (83)$$

$FA - J = \infty$

$$\begin{cases} \bar{d}_t = R(\bar{d}_{t-1})\bar{d}_{t-1} - \tilde{\kappa}\pi_t + \tilde{z} \\ \pi_t = \tilde{\alpha}d_t + \beta_F \left(R(\bar{d}_t) + R'\bar{d}_t + \gamma \frac{\partial \pi_{t+1}}{\partial d_t} \right) \pi_{t+1}. \end{cases} \quad (84)$$

where $R' \equiv \frac{\partial R(J^{-1} \sum_{j=1}^J d_{j,t})}{\partial d_t}$. We have not written these systems in matrix form, because they feature nonlinear interactions between inflation and debt. Using the budget constraint, it is easy to show the following helpful fact:

Lemma 3. Consider any of the games defined by (80)-(84), and suppose the model is *not* extremely dynamically inefficient. Then in any infinite-horizon perfect foresight equilibrium the response of inflation to debt $\frac{\partial \pi}{\partial d}$ is positive at any stable steady-state debt level.

Comparing (81) with (82), we now see two distinct common pool problems. A single government takes account of two effects of debt accumulation. Greater debt gives the central bank a stronger incentive to create inflation, reflected in the term $\gamma \frac{\partial \pi}{\partial d}$, which is positive at steady state, given Lemma 3, as long as the central bank is *moderately inflation averse* (implying $\gamma > 0$). Also, greater debt implies a higher interest rate next period, reflected in the term $R'(d)d > 0$. Both of these terms reduce inflation growth. When there are multiple countries (of equal size), each country's debt affects average debt in proportion $1/J$, so these terms are scaled down by the factor $1/J$. In the limit as $J \rightarrow \infty$, these terms disappear, as we see in (82) and (83). The reduction of these terms as J increases means that inflation growth increases with J (that is, deficit bias rises with J), which is how the common pool problem is manifested here. While in the baseline model there is a common pool problem **only** when $\gamma > 0$, with the additional effect of interest rate contagion there remains a common pool problem even if γ is zero or is mildly negative.

As before, the economies with fiscal authorities are equivalent to economies without fiscal authorities under a particular parameterization. When $\beta_F = \beta_G$ and $\alpha_{dF} = 0$ (which implies $\tilde{\alpha} = 0$) the union with country level fiscal authority collapses to the BB99, $J = \infty$ case, and the union with union-wide fiscal authority collapse to the BB99, $J = 1$ case. Thus, propositions 3(a) and 4(a) continue to hold in the extended model; and the parameterization under which fiscal authorities are neutral provides a starting point for analyzing the effects of fiscal authorities under more interesting parameterizations.

An advantage of the extended framework with a variable interest rate is that we no longer need to focus the analysis on a finite-horizon context. When comparing the regimes in our benchmark model, we focused on the path of debt for $T < \infty$, since in many cases an infinite horizon solution did not exist. In the extended framework, with interest rates that increase with debt, an infinite horizon rational expectations equilibrium exists for all regimes, even if $\beta_G > 0$ is arbitrarily small, as in Schmitt-Grohe and Uribe (2003). Therefore we can now focus on infinite-horizon rather than finite-horizon settings. In particular, our main findings can be seen by just comparing the steady states of debt and inflation across policy regimes.

To obtain analytical formulas for the steady states of the various models, we now assume a simple linear interest rate function

$$R(\bar{d}_t) = \frac{1}{\beta_S} + \delta \bar{d}_t \quad (85)$$

with an intercept term equal to the interest rate that compensates society's utility discount factor.²² For the first three cases, planner (OCCPP), BB99, $J = 1$ and BB99, $J = \infty$, the steady states of the unique rational expectations dynamics are given in the following table.

	OCCPP	BB99, $J = 1$	BB99, $J = \infty$
\bar{d}_{SS}	0	$\frac{1}{2\delta} \left(\frac{1}{\beta_G} - \frac{1}{\beta_S} \right) - \frac{\gamma}{2\delta} \frac{\partial \pi}{\partial d}$	$\frac{1}{\delta} \left(\frac{1}{\beta_G} - \frac{1}{\beta_S} \right)$
π_{SS}	$\frac{\bar{z}}{\bar{\kappa}_P}$	$\frac{\bar{z}}{\bar{\kappa}} + \left(\frac{1}{\beta_G^*} - 1 \right) \frac{\bar{d}_{SS}}{\bar{\kappa}}$	$\frac{\bar{z}}{\bar{\kappa}} + \left(\frac{1}{\beta_G} - 1 \right) \frac{\bar{d}_{SS}}{\bar{\kappa}}$

where $\frac{1}{\beta_G^*} \equiv \frac{1}{2} \left(\frac{1}{\beta_G} + \frac{1}{\beta_S} - \gamma \frac{\partial \pi}{\partial d} \right)$. Note that *moderate inflation aversion* ($\gamma > 0$) is sufficient but not necessary for $\frac{1}{\beta_G^*} < \frac{1}{\beta_G}$.

The debt bias from the additional common pool problem associated with variable interest rates can be seen by comparing the debt of an independent country with the average debt in the union. The steady state debt of a single country with an independent monetary policy (BB99, $J = 1$) is lower than that in the union (BB99, $J = \infty$) for two reasons. First, it internalizes the effect of debt on the interest rate, which decreases steady state debt by 50%.²³ Second, as previously discussed, it observes that its debt might affect inflation and therefore decreases debt further if the central bank exhibits *moderate inflation aversion*. This effect is seen in the second term in the formula for debt in the BB99, $J = 1$ case. These findings are summarized in the following results, which extend Props. 1 and 2 to the case of variable interest rates.

²²Assuming $R(\bar{d}_t) = \frac{1}{\beta_S} + \delta(\bar{d}_t - \bar{d}^*)$, where \bar{d}^* is an arbitrary target for debt, does not alter our qualitative results. Thus, for simplicity, we set the target level to zero.

²³This quantitative finding stems from the linearity $R(\bar{d}_t)$; note that since $R' = \delta$,

Prop. 5. Suppose interest rates increase with debt according to (85); also, let $T = \infty$, $J = \infty$, and $\beta_G < \beta_S$. Then steady-state debt is higher in the BB99 monetary union than it is in the OCCPP planner solution.

Prop. 6. Suppose interest rates increase with debt according to (85); also, let $T = \infty$ and $\beta_G < \beta_S$. Then steady-state debt is higher in a monetary union with $J = \infty$ than it is in a single country with an independent monetary policy if the following condition holds:

$$\left(\frac{1}{\beta_G} - \frac{1}{\beta_S} \right) + \gamma \frac{\partial \pi}{\partial d} > 0. \quad (86)$$

Moderate inflation aversion is sufficient but not necessary for (86).

We next consider the effects of introducing fiscal authorities. If $J = \infty$, and there is a fiscal authority in each country j , the relevant system (83) can be rewritten as

$$\begin{cases} \bar{d}_{SS} = \frac{\pi_{SS}(1-\beta_F/\beta_S)}{\tilde{\alpha} + \beta_F \delta \pi_{SS}} = g^{FA_j}(\pi_{SS}) \\ \pi_{SS} = \frac{\tilde{z}}{\kappa} + \frac{\bar{d}_{SS}}{\kappa} \left(\frac{1}{\beta_S} - 1 + \delta \bar{d}_{SS} \right) = f(\bar{d}_{SS}). \end{cases} \quad (87)$$

Function f is a parabola which represents the steady-state debt level consistent with the budget constraint, given steady-state inflation; it is illustrated by the blue curve in Figure 3. The inflation rate takes its minimum value, $\pi^{min} = \frac{1}{\kappa} \left(\tilde{z} - \frac{(1/\beta_S - 1)^2}{4\delta} \right)$, when debt takes the negative value $d^{min} = \frac{1 - 1/\beta_S}{2\delta} < 0$. Note that $\pi^{min} > 0$ if and only if $\tilde{z} > \frac{(1/\beta_S - 1)^2}{4\delta}$, which is a weak restriction since \tilde{z} is likely to be large. If instead $\tilde{z} < \frac{(1/\beta_S - 1)^2}{4\delta}$, then the minimum value of inflation is negative, and the two debt levels associated with $\pi_{SS} = 0$ are both negative.

The function $g^{FA_j}(\pi_{SS})$ represents the values of inflation and debt consistent with the steady-state Euler equation. This function is weakly increasing, and can be used to bound the possible values of debt: as inflation tends to zero, $g^{FA_j}(\pi_{SS}) \rightarrow 0$; and as inflation tends to infinity, $g^{FA_j}(\pi_{SS}) \rightarrow \frac{1}{\delta} \left(\frac{1}{\beta_F} - \frac{1}{\beta_S} \right)$. Note that since $\beta_F > \beta_S$, this upper bound is less than the debt level associated with the BB99, $J = \infty$ case (see the third column in the previous table). On the other hand, for negative values of inflation, $g^{FA_j}(\pi_{SS})$ goes asymptotically to $-\infty$ as $\pi_{SS} \rightarrow -\frac{\tilde{\alpha}}{\beta_F \delta}$. Geometrically, we conclude that if $\tilde{z} > \frac{(1/\beta_S - 1)^2}{4\delta}$ or if $\tilde{\alpha}$ is sufficiently large, then f and g^{FA_j} have exactly one crossing.²⁴ This crossing occurs at a point where steady state debt and inflation are both positive (both *strictly* positive, if $\beta_F < \beta_S$; but steady-state debt will be exactly zero if $\beta_F = \beta_S$). These observations imply the following comparison across regimes:

²⁴If neither of these conditions is satisfied, then f and g have three crossings, two of which occur at points where both inflation and debt are negative. These additional steady states seem uninteresting for our purposes.

Proposition 7. Suppose interest rates increase with debt according to (85); also, let $T = \infty$ and $\beta_G < \beta_F < \beta_S$. Then if $\bar{z} > \frac{(1/\beta_S - 1)^2}{4\delta}$ or if $\tilde{\alpha}$ is sufficiently large, there is a unique steady state of the FA_j , $J = \infty$ model, which has less debt than the BB99, $J = \infty$ model, and (weakly) more debt than the OCCPP planner's solution:

$$0 \leq \bar{d}_{SS}^{FA_j} < \frac{1}{\delta} \left(\frac{1}{\beta_F} - \frac{1}{\beta_S} \right) \leq \frac{1}{\delta} \left(\frac{1}{\beta_G} - \frac{1}{\beta_S} \right) = \bar{d}_{SS}^{BB99} \quad (88)$$

The leftmost inequality holds strictly if $\beta_F < \beta_S$ strictly.

This result is an infinite horizon version of Prop. 3(c): fiscal authorities at the country level decrease debt, bringing the equilibrium closer to the planner's solution.

If instead there is a single centralized fiscal authority, the relevant system of necessary conditions can be written as

$$\begin{cases} \bar{d}_{SS} = \frac{\pi_{SS}(1 - \beta_F/\beta_S - \beta_F\gamma\partial\pi/\partial\bar{d})}{\tilde{\alpha} + 2\beta_F\delta\pi_{SS}} = g^{FA}(\pi_{SS}) \\ \pi_{SS} = \frac{\tilde{\alpha}}{\kappa} + d_{SS} \left(\frac{1}{\beta_S} - 1 + \delta d_{SS} \right) = f(d_{SS}). \end{cases} \quad (89)$$

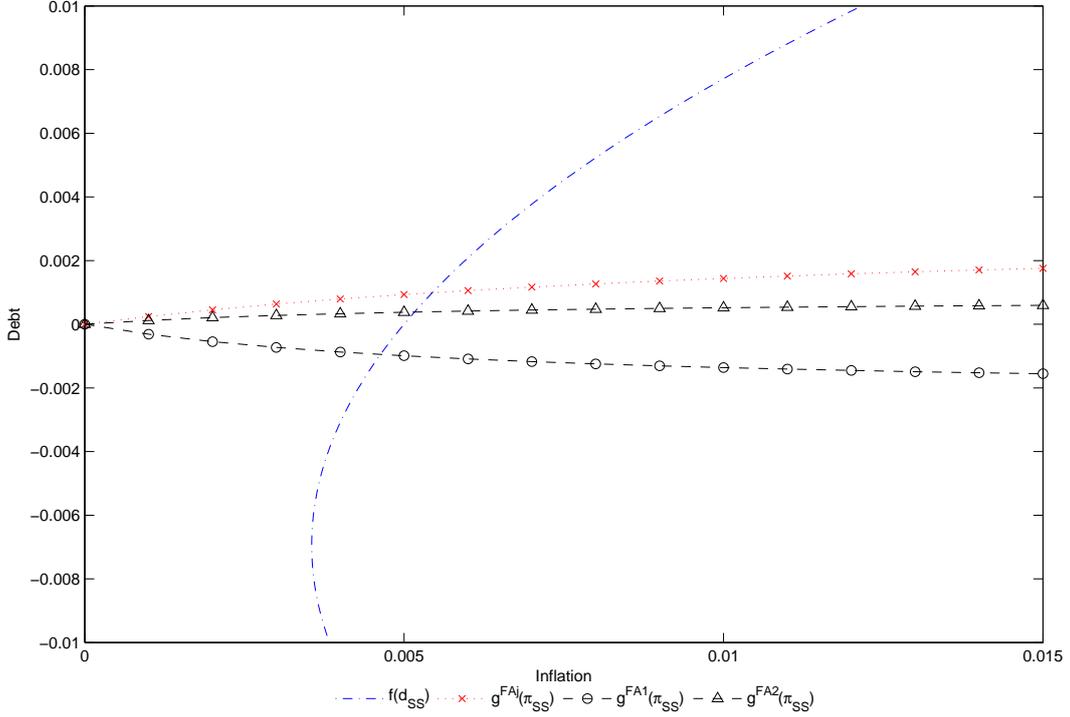
We can again bound the steady-state debt level using the function $g^{FA}(\pi_{SS})$ as we did for Prop. 7. Function g^{FA} is weakly increasing; when inflation tends to zero, $g^{FA}(\pi_{SS}) \rightarrow 0$ and when inflation tends to infinity $g^{FA}(\pi_{SS}) \rightarrow \frac{1}{2\delta} \left(\frac{1}{\beta_F} - \frac{1}{\beta_S} - \frac{\gamma\partial\pi}{\partial\bar{d}} \right)$. We must consider two possible cases:

Case 1 If $\left(\frac{1}{\beta_F} - \frac{1}{\beta_S} - \frac{\gamma\partial\pi}{\partial\bar{d}} \right) < 0$, then when inflation increases, debt approaches its lower bound (asset accumulation), and since the derivative is negative and increasing in π_{SS} , the steady state level of debt ($\bar{d}_{SS}^{FA_1}$) is negative in this case.

Case 2 $\left(\frac{1}{\beta_F} - \frac{1}{\beta_S} - \frac{\gamma\partial\pi}{\partial\bar{d}} \right) > 0$, then when inflation increases, debt approaches its upper bound, and since the derivative is positive and decreasing in π_{SS} , the steady state level of debt ($\bar{d}_{SS}^{FA_2}$) is positive. Note, however, that both the upper bound and the derivative are lower in the FA case relative for the FA_j or $g^{FA}(\pi_{SS})|_{\pi_{SS} \rightarrow \infty} < g^{FA_j}(\pi_{SS})|_{\pi_{SS} \rightarrow \infty}$ and $\frac{\partial g^{FA}(\pi_{SS})}{\partial \pi_{SS}} < \frac{\partial g^{FA_j}(\pi_{SS})}{\partial \pi_{SS}}$. Thus, we can rank the debt levels in the two fiscal authority scenarios, extending the conclusions of Prop. 4(c).

Proposition 8. Suppose interest rates increase with debt according to (85), and let $T = \infty$. Then if $\bar{z} > \frac{(1/\beta_S - 1)^2}{4\delta}$ or if $\tilde{\alpha}$ is sufficiently large, there is a unique steady state of the FA model, with less debt than the steady state of the FA_j , $J = \infty$ model.

Figure 3: Functions used to calculate debt and inflation.



Summarizing, the large monetary union of BB99 has higher debt than a large monetary union with national fiscal authorities, which in turn have higher debt than a monetary union with a centralized fiscal authority. The debt with national fiscal authorities is higher than in the social planner solution; but with a centralized fiscal authority debt may actually be too low:

$$d_{SS}^{FA1} < 0 < d_{SS}^{FA2} < d_{SS}^{FAj} < d_{SS}^{BB99}. \quad (90)$$

Figure 3 graphs the functions $f(d_{SS})$ (the blue parabola) together with $g^{FAj}(\pi_{SS})$, $g^{FA1}(\pi_{SS})$, $g^{FA2}(\pi_{SS})$. The horizontal axis represents inflation, and the vertical axis represents debt. The parameterization shown satisfies $\bar{z} > \frac{(1/\beta_S - 1)^2}{4\delta}$, so the minimum value of π_{SS} on the parabola f is positive.

5 Policy implications

Fear of moral hazard continues to hold back possible agreements among EU nations that would prevent self-fulfilling attacks on member states' sovereign debt and cross-border panics in the European banking system. The basic fear is that any mechanism capable

of preventing crises opens the door to irresponsible fiscal policies that count on future bailouts instead of ensuring long-run national budget balance. Therefore, designing an institutional framework capable of ensuring long-run fiscal discipline could be crucial for the establishment of a crisis prevention mechanism, and thus could be prove to be the key to the long-run stability of the Eurozone.

Our model points to one powerful framework for fiscal discipline: the establishment of a budgetary agency within the European Commission, mandated to ensure long-run budget balance, which for the sake of concreteness we will call the European Fiscal Authority (EFA). What exactly would the EFA do? First, it would necessarily take the form of a forecasting agency, monitoring and predicting fiscal trends in each member state. Second, it could provide advice to member governments about the likely fiscal impact of new policy proposals. In these aspects, it would be similar to the “fiscal monitoring councils” that all member states are required to establish under recent European agreements (European Council (2012)).

But most importantly, the EFA would go beyond monitoring and advice: it would exercise executive control over one or more powerful national fiscal instruments. In particular, it would set instruments with a sufficient budgetary impact to give it effective control over the path of each member state’s public debt. Our model suggests that this setup would decrease debt accumulation in three ways. First, as a technical arm of the European bureaucracy, it would be unlikely to suffer the impatience typical of elected bodies. Second, controlling just a few instruments under a mandate to maintain long-run budget balance, it would be likely to care more about the debt level than a would government charged with balancing the concerns of countless competing interest groups. Third, by taking its decisions at the union level, it would internalize the common pool problems in member states’ budget choices.

Together, according to our model, these three mechanisms imply that debt is lower in an economy with a union-wide fiscal authority than it is in an economy with national fiscal authorities (Props. 4c and 8), which in turn is lower than the debt level in the BB99 model of a large monetary union (Props. 3c and 7). Decreasing debt relative to the BB99 model is beneficial, since we have seen that it is excessively high, compared with the social planner’s solution (Props. 1 and 5). Ironically, what our model does *not* rule out is the possibility that a union-wide fiscal authority may go too far, increasing public saving beyond the social optimum. Taking this result seriously, we might conclude that some other institutional setting (country-specific fiscal authorities, or a federal fiscal government) would be superior to a union-wide fiscal authority. But our baseline model was built around a very limited and probably unrealistic form of deficit bias: as in Chari and Kehoe (2007), deficit bias arises only as a side effect of an

insufficiently inflation-averse central bank. Therefore, in Section 4 we considered an extended model in which interest rate contagion also caused deficit bias in the monetary union. Our ranking results were strengthened in this extension: moderate inflation aversion was still sufficient, but no longer necessary, to prove that a monetary union tends to accumulate more debt than a single country (Prop. 5), and we showed that a country-specific fiscal authority is insufficient, by itself, to reach the social planner's preferred debt level (Prop. 7).²⁵

Our analysis is founded on the assumption that no policy makers can commit to follow a rule. Thus, from the beginning we discard the possibility that the path of debt may be altered by imposing rules on Eurozone member states. It is unclear to us why advocates of "Fiscal Stability Union" are so eternally hopeful that rules will be respected in the future when they have been broken repeatedly in the past. This aspect of our analysis is more consistent than that of BB99, who build their model on the assumption that the central bank cannot commit to follow a monetary policy rule, but then inexplicably argue that governments could and should commit to a fiscal rule. Instead, in our model all policy decisions represent equilibrium outcomes of games between policy makers with different instruments and different preferences.²⁶ Following Rogoff (1985), we assume that institutional design may affect institutional preferences. In particular, we assume that a budgetary agency with a mandate to maintain long-run budget balance, with control over a few instruments that make this mandate feasible, will act in a debt-averse manner. This is consistent with the apparent inflation aversion of central banks that are mandated to maintain low inflation and control instruments that make low inflation feasible.²⁷

Our model addresses more explicitly the alternative possibility of establishing a "Fiscal Transfer Union", in which a federal government makes all fiscal decisions at the union level. Such a framework would internalize the common pool problem in member states' budget decisions. But unlike the EFA framework, a federal government would

²⁵Two additional sources of deficit bias that might further strengthen our results are nominal debt, and moral hazard in bailout decisions. Member states might accumulate more debt if they expect it to be inflated away, or if they expect to be bailed out, upon insolvency; such an expectation might be justified if a failure to rescue them would induce contagion on other member states' finances. These are interesting issues for future research.

²⁶In our deterministic model, there is no equilibrium role for rules. But a model of "sustainable equilibria" might allow us to incorporate the imposition of rules and punishments as an equilibrium outcome, along the lines of Chari and Kehoe (1990).

²⁷Blinder (1998) strongly advocates interpreting the Rogoff (1985) model as an argument about the effects of delegating instruments to an independent central bank, instead of the simplistic interpretation that the bank should "hire a conservative" as its governor.

be subject to the budgetary disadvantages of the democratic political process: electoral politics would make it impatient, and it would display no more debt aversion than any other government. At the same time, it would give up the advantages of the European principle of “subsidiarity”: by taking fiscal decisions at a more centralized level, it would lose local information and thus would be likely to spend less efficiently. Our model shows that this inefficient spending also tends to raise debt accumulation. For all these reasons, the EFA is likely to provide stronger fiscal discipline than a European government would. Moreover, keeping fiscal decisions as close as possible to the local level is also a way of increasing their political legitimacy. In this sense, an unelected European body charged *only* with ensuring long-run budget balance could actually produce a more democratic outcome than would an elected European government with wider fiscal powers.

If we accept that a strong fiscal disciplinary mechanism is indeed beneficial, and compatible with democracy, several further questions arise. First, is it politically feasible? Second, can effective fiscal instruments be established, in practice? And finally, which fiscal instrument(s) would be most appropriate for delegation to a hypothetical European Fiscal Agency?

In spite of the fact that delegation of fiscal instruments is not standard practice today, it does seem politically feasible in the European context. Fiscally fragile countries in the Eurozone still need assistance from the monetary authorities in order to avoid the risk of speculative attacks coming from many possible directions (contagion in response to the Cypriot bailout negotiations is only the most recent example). The monetary authorities have many possible mechanisms to protect member states against speculative attacks: one obvious possibility would be to cap the risk premium on a member state’s debt, but there are many other alternatives. However, fiscally strong European countries oppose monetary protection against speculative attacks, because they fear moral hazard: the weaker countries might fail to balance their budgets if they take ECB protection for granted.

These considerations point to a politically feasible *quid pro quo*. The European Commission could establish a budget forecasting agency, which we will call the European Fiscal Authority. The Commission could then offer all member states the option to delegate one or more fiscal instruments to the EFA, on a purely voluntary basis. The EFA would then evaluate whether the instruments proposed for delegation by a given member state give it effective control of that member state’s debt. When the EFA judges that it has been granted effective control of the member state’s debt level—including setting up the bureaucratic and legal framework for control of the proposed instruments by the EFA—the member state would become immediately eligible for

ECB protection against speculative attacks (by whatever mechanism the ECB judges appropriate).²⁸

Crucially, protection would remain contingent at all times on continuing approval from the EFA. If at any time the EFA judges that its delegated instruments are less powerful than expected, or if it judges that a member state has begun to “game the system” in some way that makes it unable to control that state’s debt level, the EFA would publicly revoke its approval of the delegated instruments (probably, but not necessarily, after adequate advance warning to the member state). The ECB would then be obliged to stop providing protection against attacks on that state’s sovereign debt. One might question whether it is credible to threaten to eliminate a member state’s protection against speculative attacks. There could be scope for moral hazard if eliminating protection of the bonds of one country caused contagion to others. But assuming that other fiscally fragile countries would be likely to participate in the EFA system, scope for contagion would be greatly mitigated.

Finally, we come to the question of which instrument(s), if any, would be appropriate for delegating control of long-term budget balance to the European Commission. Concretely, in our model, the fiscal authority actually issues each member state’s sovereign debt; the member government is then free to spend the cash proceeds. But this is an unrealistic assumption, made only for mathematical convenience (and for comparability with the BB99 paper). Most forms of public spending involve long-term projects and long-term contracts that are costly and difficult to adjust rapidly; therefore, in practice, most public spending decisions are planned long in advance, and sovereign debt issuance is typically a residual, chosen after spending and taxes to compensate any difference between the two. Indeed, formal control of debt issuance may not suffice for *de facto* control of the debt. A recent example is discussion of the issuance of “platinum coins” as a way to get around the US Congress’ legal control over the US debt level. Likewise, in recent years many countries and regions have resorted to issuing scrip or IOUs— or simply delaying payments— when for some reason they have been unable to formally issue more sovereign debt.

On the other hand, formal control of debt issuance may also not be *necessary* to achieve *de facto* control of the debt, as long as the EFA has control over some other instrument that can have a rapid and powerful effect on the current deficit. There are many possible ways to construct a powerful budgetary instrument of this sort. Probably the simplest idea is that of Gruen (1997), who proposed defining a multiplicative shift factor in the Australian tax code. He proposed applying this shift

²⁸See de Blas (7 June 2012, VoxEU), for further discussion of the establishment of an EFA.

factor to income taxes, VAT taxes, and all other types of taxes. Tax rates would take whatever complicated functional form the Australian government decided, but would subsequently be multiplied by a factor X_t , which would initially be set to one but would thereafter be adjusted by an independent fiscal authority to ensure control of the debt level.

Another instrument is implicit in the analysis of Gomes (2011), who shows that public sector wages should optimally be state-contingent, rising in times of fiscal plenty and falling when the budget is tight. Across-the-board shifts in public labor compensation would have a powerful budgetary impact, and could in principle be performed very quickly (particularly if they are a systematic aspect of public contracts, instead of being an *ad hoc* crisis response, as they have often been in practice). Costain and de Blas (2012a,b) go a step further and point out that *all public sector prices* could be made effectively state-contingent by budgeting them in an alternative currency, the value of which would be determined by the fiscal authority. In the European context, this could be a way of reestablishing some of the nominal flexibility that is usually assumed lost upon joining a monetary union.²⁹ Finally, it is increasingly common that pension systems automatically take into account demographic adjustment factors in determining retirement ages or benefit levels. Additional adjustment factors related to long-term budget trends would be another potentially powerful lever that could be delegated to an independent fiscal authority.³⁰

Each of these instruments has different political, economic, and distributional effects; ultimately it should be a democratic decision for each member state whether or not to participate in the EFA mechanism, and if so, which instrument(s) it prefers to delegate to the fiscal authority.³¹ All other fiscal instruments would remain under the control of the member government, consistent with the European principle of “subsidiarity”. The only decision in the hands of the fiscal authority itself would be the technical and quantitative question of what setting of its delegated instrument is consistent with long-run budget balance under its forecasts, given the fiscal decisions of the member government.

²⁹Dornbusch (1997) discusses a historical precedent for the idea of establishing a unit of account different from the medium of exchange: Brazil’s successful disinflation program of 1994.

³⁰A panel of experts appointed by the Spanish government recently recommended the establishment of a “Factor de Revalorización Anual” that would regularly update pensions and/or pension contributions in response to any persistent differences between the two; see El País (21 June 2013).

³¹Whether or not the analysis in this paper is robust to substituting actual debt issuance by another instrument like those discussed here is an important question which we hope to pursue in future work.

6 Conclusions

This paper has analyzed the potential for the delegation of fiscal instruments to offset systematic deficit bias in the fiscal decisions of democratic governments, with particular attention to the context of a monetary union. That is, we ask to what extent deficit bias could be reduced by a fiscal authority that is independent of government and has a mandate for long-run budget balance, just as independent, inflation-averse central banks have helped reduce inflation bias. We follow the simple modeling strategy of Rogoff (1985), characterizing differences across institutions by different weight parameters in their objective functions. First, we assume electoral politics induces impatience in democratic institutions, relative to society's discount rate; second, we assume that mandating an institution to pursue a single, simple, quantitative objective skews its preferences in favor of that objective, relative to the social welfare function.

This parsimonious treatment of institutions allows us to derive strong analytical results comparing equilibrium outcomes across different institutional configurations. We focus on perfect-foresight equilibria, in order to shed light on systematic biases. We start from a baseline institutional framework, based on Beetsma and Bovenberg (1999), in which a single central bank controls inflation for all the regions in a monetary union, while region-specific democratic governments make all fiscal decisions in each region. We compare several alternative configurations: (1) in each region, debt is issued by a debt-averse regional fiscal authority, leaving all other fiscal decisions up to the regional governments; (2) debt is issued for each region by a union-wide debt-averse fiscal authority, leaving all other fiscal decisions up to the regional governments; (3) the regional governments are replaced by a single, union-wide government. We show (a) per capita debt is higher in the baseline scenario than it would be for a single country with an independent monetary policy; (b) debt is excessive in the baseline scenario, relative to a social planner's solution; (c) debt is lower under region-specific fiscal authorities than it is in the baseline scenario, and (d) it is lower still under a single, union-wide fiscal authority. Under the scenario with a union-wide government (but no fiscal authority), there is a tendency to accumulate less debt than in the baseline scenario, because the common-pool problems in deficit choice are internalized, but the other debt-avoiding properties of the fiscal authority are lost, and public spending attains less bang for the buck due to a loss of informational efficiency.

Going beyond the model, we have discussed the role that fiscal delegation might play in resolving the ongoing Eurozone crisis, where a disciplined fiscal regime is a crucial counterpart (both economically and politically) to most of the monetary and financial mechanisms currently under debate. A European Fiscal Authority controlling

at least one sufficiently powerful fiscal instrument in a member state could guarantee that state's long-run budget balance. The member state itself would decide which instrument to delegate, while the EFA would evaluate whether it is "sufficiently powerful". Delegation to the EFA would be attractive if it made member states eligible for ECB protection against speculative attacks; but even without such a guarantee it could be attractive as a way of improving fiscal credibility and lowering risk premia. Therefore we have stressed that these institutions could be constructed in a voluntary, step-by-step fashion. As long as fears of moral hazard persist, peripheral countries can do little to achieve a union-wide agreement that would protect them against any future shocks to the Eurozone. Reforming their fiscal institutions—possibly unilaterally—is one way peripheral countries could jumpstart the negotiations for such an agreement.

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7 Appendix: Proofs.

Proof of Proposition 1. We will use the superscript P to refer to the social planner solution. Analogous quantities without superscripts refer to the BB99 model.

Given our linear-quadratic model, in which the only state variable is the debt level, we expect a linear solution of the form

$$\pi_t = C_t^\pi d_{t-1} + \epsilon_t^\pi \quad (91)$$

$$d_t = C_t^d d_{t-1} + \epsilon_t^d \quad (92)$$

where C_t^π , ϵ_t^π , C_t^d , and ϵ_t^d are coefficients to be determined, which will vary with time in a finite-horizon solution. To show that the proposition is true, it will suffice to show that for all times $t \in \{0, 1, \dots, T - 1\}$, the coefficients satisfy

$$C_t^d > C_t^{d,P} > 0, \quad (93)$$

$$\epsilon_t^d \geq \epsilon_t^{d,P} \geq 0. \quad (94)$$

Positivity of the coefficients in (93)-(94) implies that if $d_{-1} > 0$, then $d_t > 0$ for all $t \in \{0, 1, \dots, T - 1\}$. The ranking of the coefficients in (93)-(94) then allows us to conclude that if $d_t \geq d_t^P$ for some t , then $d_{t+1} > d_{t+1}^P$ in the next period, and likewise in future periods.

In each policy regime, the budget constraint implies that the coefficients for debt and inflation are linked by

$$C_t^d = R - \tilde{\kappa} C_t^\pi, \quad \epsilon_t^d = \tilde{z} - \tilde{\kappa} \epsilon_t^\pi; \quad (95)$$

$$C_t^{d,P} = R - \tilde{\kappa}_P C_t^{\pi,P}, \quad \epsilon_t^{d,P} = \tilde{z} - \tilde{\kappa}_P \epsilon_t^{\pi,P}. \quad (96)$$

Therefore (93)-(94) hold if and only if

$$\tilde{C}_t^\pi \equiv \tilde{\kappa} C_t^\pi < \tilde{\kappa}_P C_t^{\pi,P} \equiv \tilde{C}_t^{\pi,P} < R, \quad (97)$$

$$\tilde{\epsilon}_t^\pi \equiv \tilde{\kappa} \epsilon_t^\pi \leq \tilde{\kappa}_P \epsilon_t^{\pi,P} \equiv \tilde{\epsilon}_t^{\pi,P} \leq \tilde{z}, \quad (98)$$

for all $t \in \{0, 1, \dots, T-1\}$.

In the last period, the budget constraint $d_T = 0$ implies

$$\tilde{C}_T^\pi = \tilde{C}_T^{\pi,P} = R, \quad (99)$$

$$\tilde{\epsilon}_T^\pi = \tilde{\epsilon}_T^{\pi,P} = \tilde{z}. \quad (100)$$

In previous periods, we can evaluate the unknown coefficients by plugging (91)-(92) into the inflation dynamics. For the BB99 regime (with $J = \infty$), we plug the linear solution into (37) to evaluate \tilde{C}_t^π and $\tilde{\epsilon}_t^\pi$, given \tilde{C}_{t+1}^π and $\tilde{\epsilon}_{t+1}^\pi$:

$$\tilde{C}_t^\pi = \frac{\beta_G R^2 \tilde{C}_{t+1}^\pi}{1 + \beta_G R \tilde{C}_{t+1}^\pi}, \quad (101)$$

$$\tilde{\epsilon}_t^\pi = \frac{\beta_G R \tilde{\epsilon}_{t+1}^\pi + \tilde{z} \beta_G R \tilde{C}_{t+1}^\pi}{1 + \beta_G R \tilde{C}_{t+1}^\pi}. \quad (102)$$

Under the social planner solution, the inflation dynamics are instead given by (17). Therefore the coefficients evolve according to

$$\tilde{C}_t^{\pi,P} = \frac{\beta_S R^2 \tilde{C}_{t+1}^{\pi,P}}{1 + \beta_S R \tilde{C}_{t+1}^{\pi,P}}, \quad (103)$$

$$\tilde{\epsilon}_t^{\pi,P} = \frac{\beta_S R \tilde{\epsilon}_{t+1}^{\pi,P} + \tilde{z} \beta_S R \tilde{C}_{t+1}^{\pi,P}}{1 + \beta_S R \tilde{C}_{t+1}^{\pi,P}}. \quad (104)$$

Now note that (103) implies $\tilde{C}_t^{\pi,P} < R$ for any $t < T$. Together with (101), note that $\tilde{C}_t^\pi < \tilde{C}_t^{\pi,P}$ holds if and only if

$$\beta_G R^2 \tilde{C}_{t+1}^\pi \left(1 + \beta_S R \tilde{C}_{t+1}^{\pi,P}\right) < \beta_S R^2 \tilde{C}_{t+1}^{\pi,P} \left(1 + \beta_G R \tilde{C}_{t+1}^\pi\right),$$

which is true if and only if

$$\beta_G \tilde{C}_{t+1}^\pi < \beta_S \tilde{C}_{t+1}^{\pi,P}.$$

Therefore, starting from $\tilde{C}_T^\pi = \tilde{C}_T^{\pi,P} = R$ and using $\beta_G < \beta_S$, we conclude by mathematical induction that

$$\tilde{C}_t^\pi < \tilde{C}_t^{\pi,P} < R \quad \text{for all } t \in \{0, 1, \dots, T-1\}. \quad (105)$$

It is also easily shown by induction that

$$\tilde{C}_t^{\pi,P} \tilde{\epsilon}_t^\pi = \tilde{C}_t^\pi \tilde{\epsilon}_t^{\pi,P} \quad (106)$$

for all $t \leq T$. Now, using (102) and (104), note that $\tilde{\epsilon}_t^\pi < \tilde{\epsilon}_t^{\pi,P}$ if and only if

$$\beta_G R \left(1 + \beta_S R \tilde{C}_{t+1}^{\pi,P}\right) \left(\epsilon_{t+1}^\pi + \tilde{z} \tilde{C}_{t+1}^\pi\right) < \beta_S R \left(1 + \beta_G R \tilde{C}_{t+1}^\pi\right) \left(\epsilon_{t+1}^{\pi,P} + \tilde{z} \tilde{C}_{t+1}^{\pi,P}\right)$$

which, using (106), is true if and only if

$$\beta_G \left(\epsilon_{t+1}^\pi + \tilde{z} \tilde{C}_{t+1}^\pi\right) < \beta_S \left(\epsilon_{t+1}^{\pi,P} + \tilde{z} \tilde{C}_{t+1}^{\pi,P}\right). \quad (107)$$

Starting from $0 < \epsilon_T^\pi = \epsilon_T^{\pi,P} = \tilde{z}$ and using $\beta_G < \beta_S$ and (105), it is easy to verify by induction that $0 < \epsilon_t^\pi < \epsilon_t^{\pi,P}$ for all $t < T$.

Finally, starting from $\epsilon_T^{\pi,P} = \tilde{z}$, notice that $\epsilon_{T-1}^{\pi,P}$ is bounded above by \tilde{z} :

$$\tilde{\epsilon}_t^{\pi,P} = \beta_S R \frac{\tilde{\epsilon}_{t+1}^\pi + \tilde{z}_t C_{t+1}^{\pi,P}}{1 + C_{t+1}^{\pi,P}} \leq \tilde{z} \quad \text{if } \tilde{\epsilon}_{t+1}^\pi \leq \tilde{z} \quad (108)$$

And by induction we conclude

$$0 \leq \tilde{\epsilon}_t^\pi \leq \tilde{\epsilon}_t^{\pi,P} \leq \tilde{z} \quad \text{for all } t \in \{0, 1, \dots, T-1\}. \quad (109)$$

QED.

Proof of Prop. 3(b). The matrix governing the dynamics of (66) has trace $T_F = R + \frac{1+\tilde{\alpha}\tilde{\kappa}}{\beta_F R}$ and determinant β_F^{-1} . The associated eigenvalues are

$$\rho_+ \equiv \frac{T_F + \sqrt{T_F^2 - 4/\beta_F}}{2}, \quad \rho_- \equiv \frac{T_F - \sqrt{T_F^2 - 4/\beta_F}}{2}. \quad (110)$$

Note that $T_F^2 - 4/\beta_F > \left(R - \frac{1}{\beta_F R}\right)^2 > 0$; therefore both eigenvalues are real. Therefore we also have

$$\rho_+ > \frac{R + \frac{1+\tilde{\alpha}\tilde{\kappa}}{\beta_F R} + \left(R - \frac{1}{\beta_F R}\right)}{2} = R + \frac{\tilde{\alpha}\tilde{\kappa}}{2\beta_F R} > 1. \quad (111)$$

Since $\sqrt{T_F^2 - 4/\beta_F}$ is real and is less than T_F , ρ_- is positive. If we regard this eigenvalue as a function of T_F , then its derivative is

$$\frac{d\rho_-}{dT_F} = \frac{1}{2} - \frac{T_F}{2\sqrt{T_F^2 - 4/\beta_F}}. \quad (112)$$

Since $\sqrt{T_F^2 - 4/\beta_F}$ is real and is less than T_F , $\frac{d\rho_-}{dT_F}$ is negative.

Note that $\rho_- = 1$ if $\sqrt{T_F^2 - 4/\beta_F} = T_F - 2$. Squaring both sides, this requires $T_F^2 - 4/\beta_F = T_F^2 - 4T_F + 4$, which implies $T_F = \beta_F^{-1} + 1$. Since ρ_- decreases with T_F , we conclude that $\rho_- < 1$ as long as $T_F > \beta_F^{-1} + 1$.

Using the definition of T_F , we see that $\rho_- = 1$ exactly if $\alpha_{dF} = 0$ and $\beta_F R = 1$. More generally, $\rho_- < 1$ strictly as long as

$$\frac{1}{\beta_F R} + \frac{\tilde{\alpha}\tilde{\kappa}}{\beta_F R} + R - \frac{1}{\beta_F} - 1 > 0, \quad (113)$$

that is, as long as

$$\alpha_{dF} > \frac{1}{\tilde{\kappa}} \left(\frac{\alpha_{\pi C} \alpha_{gF}}{\alpha_{gG} + \kappa \alpha_{gC}} \right) [R + \beta_F R - \beta_F R^2 - 1]. \quad (114)$$

Thus there is one stable and one unstable eigenvalue, implying that a unique PFE exists, as long as (114) is satisfied. **QED.**

Proof of Lemmas 1 and 2. As in Prop. 1, we expect a linear solution of the form (91)-(92). For both lemmas, the backwards updating equation for the coefficient C_t^π can be written in the form

$$C_t^\pi = h(C_{t+1}^\pi) \equiv \frac{R}{\tilde{\kappa}} \left(\frac{\tilde{\alpha} + \beta C_{t+1}^\pi (R + \gamma C_{t+1}^\pi)}{\tilde{\kappa}^{-1} + \tilde{\alpha} + \beta C_{t+1}^\pi (R + \gamma C_{t+1}^\pi)} \right),$$

where we set $\beta = \beta_G$ and $\tilde{\alpha} = 0$ for the BB99, $J = 1$ case, and $\beta = \beta_F$ for the FA case.

Assuming $\gamma > 0$, $h(C)$ is an increasing, concave function for $C \geq 0$, bounded above by $R/\tilde{\kappa}$, which crosses the 45° line at exactly one positive C . (If $\tilde{\alpha} = 0$, it also crosses at $C = 0$, where it has slope greater than one, using the assumption of *moderate impatience*.) Therefore, working backwards from $C_T^\pi = R/\tilde{\kappa}$, iteration on $h(C)$ converges to a positive limit $C^{\pi*}$.

If instead $\gamma < 0$, then $h(C)$ is positive, increasing, concave, and bounded above by $R/\tilde{\kappa}$ on the interval $(0, \frac{R}{2|\gamma|})$. If $\tilde{\alpha} > 0$, then $h(0) > 0$; if instead $\tilde{\alpha} = 0$, then $h(0) = 0$ and $h'(0) > 1$ (using the assumption of *moderate impatience*). The function $h(C)$ achieves a maximum at $\frac{R}{2|\gamma|}$, beyond which it decreases and eventually becomes negative. Therefore, if $\frac{R}{\tilde{\kappa}} < \frac{R}{2|\gamma|}$, $h(C)$ crosses the 45° line at exactly one $C \in (0, \frac{R}{2|\gamma|})$. Therefore, if $\gamma > -\frac{\tilde{\kappa}}{2}$, then working backwards from $C_T^\pi = R/\tilde{\kappa}$, iteration on $h(C)$ converges to a positive limit $C^{\pi*}$. **QED.**

Proof of Propositions 3(c) and 4(c). We will use the superscripts F_j and F to refer to the models with a country-specific or union-wide fiscal authority, respectively; quantities without superscripts refer to the BB99 model. Using this notation, Props. 3(c) and 4(c) can be stated jointly as follows.

Proposition. Suppose $J = \infty$, $T < \infty$, $\beta_F > \beta_G$, $\alpha_{dF} > 0$, $\gamma_F > 0$, and $\bar{z}_t = \bar{z}$ is a constant. Then starting from the same positive initial average debt level $\bar{d}_{-1} = \bar{d}_{-1}^{F_j} = \bar{d}_{-1}^F > 0$, equilibrium implies

$$\bar{d}_t > \bar{d}_t^{F_j} > \bar{d}_t^F \quad \text{for all } t \in \{0, 1, \dots, T-1\}. \quad (115)$$

Proof. As in Prop. 1, we expect a linear solution of the form (91)-(92). To prove the proposition, we will show that for all $t \in \{0, 1, \dots, T-1\}$, the coefficients satisfy

$$C_t^d > C_t^{d, F_j} > C_t^{d, F} > 0, \quad (116)$$

$$\epsilon_t^d \geq \epsilon_t^{d, F_j} \geq 0, \quad (117)$$

$$\epsilon_t^{d, F_j} \geq \epsilon_t^{d, F}, \quad (118)$$

which will be true if and only if, for all $t \in \{0, 1, \dots, T-1\}$,

$$C_t^\pi < C_t^{\pi, F_j} < C_t^{\pi, F} < \frac{R}{\tilde{\kappa}}, \quad (119)$$

$$\epsilon_t^\pi \leq \epsilon_t^{\pi, F_j} \leq \frac{\bar{z}}{\tilde{\kappa}}, \quad (120)$$

$$\epsilon_t^{\pi, F_j} \leq \epsilon_t^{\pi, F}. \quad (121)$$

In the last period, the budget constraint $d_T = 0$ implies

$$C_T^\pi = C_T^{\pi, F_j} = C_T^{\pi, F} = \frac{R}{\tilde{\kappa}}, \quad (122)$$

$$\epsilon_T^\pi = \epsilon_T^{\pi, F_j} = \epsilon_T^{\pi, F} = \frac{\bar{z}}{\tilde{\kappa}}. \quad (123)$$

Now for each policy regime, we can evaluate the coefficients by plugging (91)-(92) into the inflation dynamics. For the BB99 regime (with $J = \infty$), we have

$$C_t^\pi = \frac{\beta_G R^2 C_{t+1}^\pi}{1 + \tilde{\kappa} \beta_G R C_{t+1}^\pi}, \quad (124)$$

$$\epsilon_t^\pi = \frac{\beta_G R \epsilon_{t+1}^\pi + \bar{z} \beta_G R C_{t+1}^\pi}{1 + \tilde{\kappa} \beta_G R C_{t+1}^\pi}. \quad (125)$$

In the regime with national fiscal authorities (assuming $J = \infty$), the inflation dynamics are given by (58), which implies that the coefficients are governed by

$$C_t^{\pi, F_j} = \frac{R(\tilde{\alpha} + \beta_F R C_{t+1}^{\pi, F_j})}{1 + \tilde{\kappa}(\tilde{\alpha} + \beta_F R C_{t+1}^{\pi, F_j})}, \quad (126)$$

$$\epsilon_t^{\pi, F_j} = \frac{\beta_F R \epsilon_{t+1}^{\pi, F_j} + \bar{z}(\tilde{\alpha} + \beta_F R C_{t+1}^{\pi, F_j})}{1 + \tilde{\kappa}(\tilde{\alpha} + \beta_F R C_{t+1}^{\pi, F_j})}. \quad (127)$$

With a single union-wide authority (regardless of J), the inflation dynamics are given by (64). Using the conjectured linear solution, we substitute $\frac{\partial \pi_{t+1}}{\partial d_t} \equiv C_{t+1}^{\pi, F}$ in (64). The evolution of the coefficients is then:

$$C_t^{\pi, F} = \frac{R(\tilde{\alpha} + \beta_F RC_{t+1}^{\pi, F} + \beta_F \gamma_F (C_{t+1}^{\pi, F})^2)}{1 + \tilde{\kappa}(\tilde{\alpha} + \beta_F RC_{t+1}^{\pi, F} + \beta_F \gamma_F (C_{t+1}^{\pi, F})^2)}, \quad (128)$$

$$\epsilon_t^{\pi, F} = \frac{(\beta_F R + \beta_F \gamma_F C_{t+1}^{\pi, F}) \epsilon_{t+1}^{\pi, F} + \tilde{z}(\tilde{\alpha} + \beta_F RC_{t+1}^{\pi, F} + \beta_F \gamma_F (C_{t+1}^{\pi, F})^2)}{1 + \tilde{\kappa}(\tilde{\alpha} + \beta_F RC_{t+1}^{\pi, F} + \beta_F \gamma_F (C_{t+1}^{\pi, F})^2)}. \quad (129)$$

Given (124) and (126), note that $C_t^\pi < C_t^{\pi, F_j}$ if and only if

$$\beta_G RC_{t+1}^\pi \left(1 + \tilde{\kappa}(\tilde{\alpha} + \beta_F RC_{t+1}^{\pi, F_j})\right) < \left(\tilde{\alpha} + \beta_F RC_{t+1}^{\pi, F_j}\right) (1 + \tilde{\kappa} \beta_G RC_{t+1}^\pi),$$

which is true if and only if

$$\beta_G RC_{t+1}^\pi < \tilde{\alpha} + \beta_F RC_{t+1}^{\pi, F_j}.$$

Using $\beta_G < \beta_F$ and/or $\tilde{\alpha} > 0$, and starting from $C_T^\pi = C_T^{\pi, F_j} = R/\tilde{\kappa}$, we conclude by induction that $C_t^\pi < C_t^{\pi, F_j}$ for all $t < T$. Assuming *moderate inflation aversion* ($\gamma_F > 0$), a similar argument comparing (126) and (128) shows that $C_t^{\pi, F_j} < C_t^{\pi, F}$ for all $t < T$. Also, (128) shows that $C_t^{\pi, F} < R/\tilde{\kappa}$. Summarizing, if $\gamma_F > 0$, and if $\beta_F \geq \beta_G$ and $\tilde{\alpha} \geq 0$ with at least one strict inequality, then

$$0 < C_t^\pi < C_t^{\pi, F_j} < C_t^{\pi, F} < \frac{R}{\tilde{\kappa}} \quad \text{for all } t \in \{0, 1, \dots, T-1\}. \quad (130)$$

These arguments ranking the C^π coefficients under the three regimes are illustrated in Fig. 4, which shows that the graph of (128) lies everywhere above (126), which in turn lies above (124). The coefficients at each time are derived by iterating backwards on these difference equations starting from $C_T^\pi = C_T^{\pi, F_j} = C_T^{\pi, F} = R/\tilde{\kappa}$, converging towards the points where the graphs cross the 45° line; the ranking across coefficients thus holds at all times t and also in the limit as $t \rightarrow -\infty$.

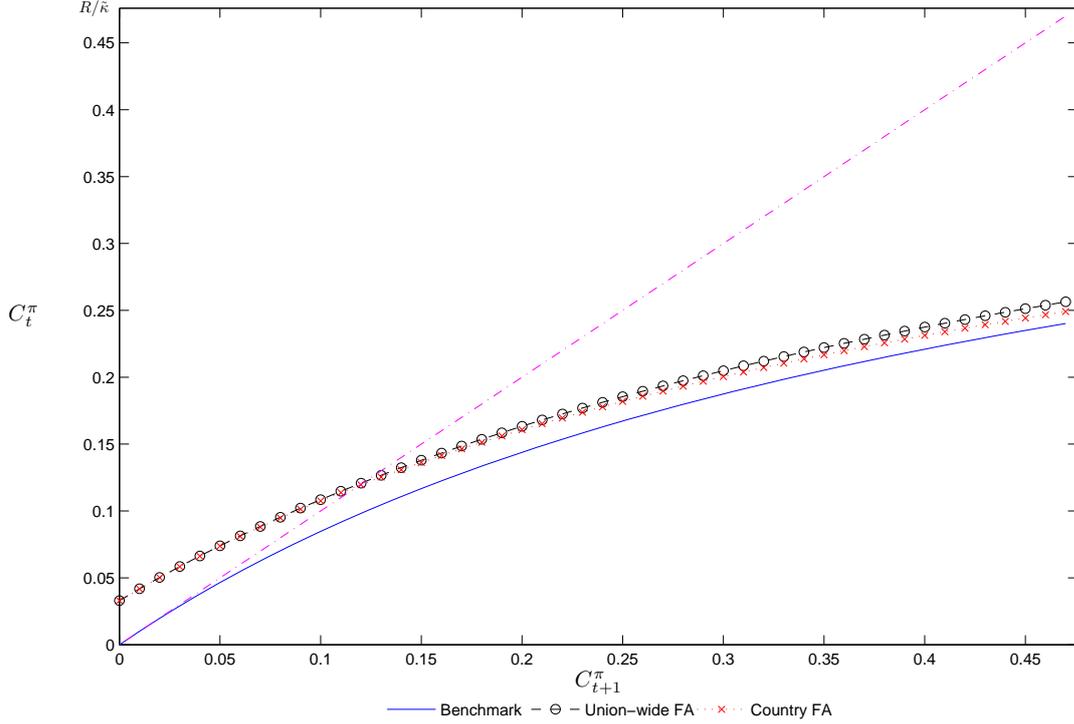
Next, note that in the final period, $\epsilon_T^{\pi, F_j} = \tilde{z}/\tilde{\kappa}$. From here we can work backwards to bound ϵ_t^{π, F_j} at all times:

$$\epsilon_{T-1}^{\pi, F_j} = \frac{\beta_F R \tilde{z}/\tilde{\kappa} + \tilde{z} \Theta_{t+1}}{1 + \tilde{\kappa} \Theta_{t+1}} \leq \frac{\tilde{z}}{\tilde{\kappa}},$$

where $\Theta_{t+1} \equiv \tilde{\alpha} + \beta_F RC_{t+1}^{\pi, F_j}$. By induction, we also have $\epsilon_t^{\pi, F_j} \leq \tilde{z}/\tilde{\kappa}$ at all earlier t .

Now, if $\beta_F = \beta_G$ and $\tilde{\alpha} = 0$, then (127) is equivalent to (125) (because then $C_{t+1}^\pi = C_{t+1}^{\pi, F_j}$). Differentiating, we find that the right-hand side of (127) is increasing in β_F , and it is also weakly increasing in $\tilde{\alpha}$ and C_{t+1}^{π, F_j} if and only if $\tilde{z} - \tilde{\kappa} \beta_F R \epsilon_{t+1}^{\pi, F_j} \geq 0$,

Figure 4: Example: Inflation coefficients.



Note. Difference equation, calculating coefficient C_t^π , given C_{t+1}^π . Graph shows that coefficients for BB99 model (blue line) always lie below the coefficients for the model with region-specific fiscal authorities (red crosses), which in turn lie below the coefficients for the model with a single union-wide fiscal authority (black circles), as stated in equation (119).

which is true since $\epsilon^{\pi, F_j} \leq \tilde{z}/\tilde{\kappa}$. Therefore, starting from $\epsilon_T^\pi = \epsilon_T^{\pi, F_j}$ and $C_T^\pi = C_T^{\pi, F_j}$, we can reason backwards using (125) and (127) to show that $\epsilon_{T-1}^\pi \leq \epsilon_{T-1}^{\pi, F_j} \leq \tilde{z}/\tilde{\kappa}$. But also, the right-hand side of (127) is increasing in $\epsilon_{t+1}^{\pi, F_j}$. Thus we can work backwards over all $t < T$, to conclude that if $\beta_F \geq \beta_G$ and $\tilde{\alpha} \geq 0$, and $\tilde{z}_t = \tilde{z}$ is a constant, then

$$\epsilon_t^\pi \leq \epsilon_t^{\pi, F_j} \leq \frac{\tilde{z}}{\tilde{\kappa}} \quad \text{for all } t \in \{0, 1, \dots, T-1\}. \quad (131)$$

Finally, when $\gamma_F = 0$, (129) is equivalent to (127). Differentiating (129) and performing some simplifications, it can be shown that $\epsilon_t^{\pi, F}$ is increasing in γ_F :

$$\frac{\partial \epsilon_t^{\pi, F}}{\partial \gamma_F} = \frac{\tilde{z}_t \beta_F (C_{t+1}^{\pi, F})^2 + \beta_F C_{t+1}^{\pi, F} \epsilon_{t+1}^{\pi, F} + \tilde{\alpha} \tilde{\kappa} \beta_F C_{t+1}^{\pi, F} \epsilon_{t+1}^{\pi, F}}{\left[1 + \tilde{\kappa} (\tilde{\alpha} + \beta_F R C_{t+1}^{\pi, F} + \beta_F \gamma_F (C_{t+1}^{\pi, F})^2)\right]^2} > 0. \quad (132)$$

It is also easy to see that if $\gamma_F \geq 0$, then $\epsilon_t^{\pi, F}$ is increasing in $\epsilon_{t+1}^{\pi, F}$. Therefore, starting from $\epsilon_T^{\pi, F_j} = \epsilon_T^{\pi, F} = \tilde{z}/\tilde{\kappa}$, we can work backwards to show that under moderate inflation aversion,

$$\epsilon_t^{\pi, F_j} \leq \epsilon_t^{\pi, F} \quad \text{for all } t \in \{0, 1, \dots, T-1\}. \quad (133)$$

But notice that in contrast to (131), we can *not* conclude that $\epsilon_t^{\pi, F}$ is less than $\bar{z}/\bar{\kappa}$. Instead, starting from $\epsilon_T^{\pi, F} = \bar{z}/\bar{\kappa}$, (128) implies $\epsilon_{T-1}^{\pi, F} > \bar{z}/\bar{\kappa}$ if $\beta_F(R + \gamma_F C_T^{\pi, F}) > 1$.

Now, (130) implies (116); (131) implies (117). Thus, starting from the same positive debt level $d_{-1} = d_{-1}^{F_j} > 0$, debt will remain higher in the BB99 monetary union than it would be under national fiscal authorities; and in both cases, it will be positive:

$$\bar{d}_t > \bar{d}_t^{F_j} > 0 \quad \text{for all } t \in \{0, 1, \dots, T-1\}. \quad (134)$$

Next, (133) implies (118). If \bar{d}_t^F always remains positive, then (116) and (118) imply that $\bar{d}_t^{F_j} > \bar{d}_t^F$ at all times; if instead \bar{d}_t^F eventually becomes negative, then we have $\bar{d}_t^{F_j} > \bar{d}_t^F$ since $\bar{d}_t^{F_j}$ is positive. Thus we have ranked the debt levels across the three regimes at all times. **QED.**

Proof of Lemma 3. Considering any one of the systems (80)-(84), let $\pi = \Pi(d)$ be the equilibrium relation between debt and inflation, and let $d' = B(d)$ be the gross borrowing function (equilibrium debt next period, as a function of debt this period). The budget constraint implies that these functions are related as follows:

$$B(d) = R(d)d - \bar{\kappa}\Pi(d) + \bar{z} \quad (135)$$

($\bar{\kappa}$ is replaced by $\bar{\kappa}^P$ in the social planner version (80)). Differentiating and rearranging, we must have

$$\Pi'(d) = \bar{\kappa}^{-1} (R(d) + R'(d)d - B'(d)). \quad (136)$$

Note that a *stable* steady state d^* is a point $d^* = B(d^*)$ characterized by $B'(d^*) < 1$. Note that for any d , total interest payments are $(R(d) - 1)d$. Therefore interest payments are increasing in debt if and only if $R'(d)d + R(d) - 1 > 0$. Thus, if $R'(d^*)d^* + R(d^*) < 1$ at the steady state debt level d^* , a permanent increase in debt would imply permanently lower interest payments (equivalently, a permanent *decrease* in asset holdings would imply permanently *higher* interest *earnings*). We have called this *extreme dynamic inefficiency*, since an economy in this steady state could permanently consume more by immediately eating up some of its savings (or by immediately increasing its debt).

At a stable steady state where the economy is *not* extremely dynamically inefficient, we have $B'(d^*) < 1 < R'(d^*)d^* + R(d^*)$, which implies $\Pi'(d) > 0$. **QED.**