An Equilibrium Theory of Credit Rating

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Abstract

We develop an equilibrium theory of credit rating. By influencing rational creditors, ratings affect firms' probability of default, which in turn affects ratings. In equilibrium, credit rating is pro-cyclical and magnifies underlying market conditions. Moreover, biased incentives of credit rating agencies are ultimately self-defeating – a bias in favor of issuers raises the incidence of default.

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1 Introduction

Credit rating agencies (CRAs) have faced heavy fire in the wake of the 2008 financial crisis. It was first argued that CRAs fueled the build-up to the crisis by being too lenient, particularly

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regarding mortgage-backed securities and other 'new' financial products.[1] CRAs were then accused of worsening the turmoil after the crisis erupted and, most prominently, of downgrading sovereign debt with no clear deterioration in fundamentals to justify these downgrades.[2] CRAs themselves, on the other hand, argue that their agenda is simply to provide investors with reliable information. This begs the following questions: why – and how – do ratings' standards and impact vary with macroeconomic contingencies?

To address these issues, we develop an equilibrium model of credit rating with three key features: (i) ratings which affect the performance of rated securities, (ii) strategic CRAs aiming at accurate predictions, and (iii) rational expectations on the part of investors. A firm seeks to roll over its debt in order to meet its short-term liquidity needs. By providing information to investors, ratings affect the firm's probability of default. This in turn influences the CRA's optimal rating decision – causing feedback effects between ratings and firms' chances of default. We show that when aggregate liquidity is easy, ratings are inflated and on average decrease the incidence of default. By contrast, when liquidity is tight, ratings are deflated and on average increase the incidence of default. Moreover, biased incentives on the part of the CRA are ultimately self-defeating. In particular, a bias in favor of issuers raises the incidence of default.

The model works as follows. A credit rating agency publicly announces its chosen rating of a firm (or sovereign).[3] A mass of creditors then decide whether to roll over or liquidate their loan, based on the rating of the CRA as well as their own private information regarding the

[1] See, for example, U.S. Senate Permanent Subcommittee on Investigations, 2011.

[2] For example, in a joint letter 6 May 2010, Chancellor Angela Merkel and President Nicolas Sarkozy write that 'The decision of a rating agency to downgrade the rating of the Greek debt even before the authorities' programme and amount of the support package were known must make us ponder the rating agencies' role in propagating crises.' See also Paul Krugman's New York Times article 'Berating the raters', April 26, 2010.

[3] While we frame our study in general terms, our model arguably fits best the example of sovereign refinancing.

firm's chances of success.[4] Liquidating involves a partial loss, but allows creditors to receive immediate payment, and avoid the risk of a later default due to coordination failure.

Investors discount delayed payments according to their immediate need for cash: when liquidity is tight, investors discount future payments more. The CRA, on the other hand, is driven by the desire to preserve its track record by making correct predictions, i.e. by giving good/bad ratings to firms that later succeed/default.

Ratings affect investors' decisions, which then determine the firm's probability of default. This, in turn, influences the rating of the CRA.

We first examine ratings' equilibrium impact. Tight liquidity makes investors more reluctant to roll over their loan, while it is of no *direct* concern to the CRA.[5] Thus, when liquidity is tight, investors perceive CRAs as lenient relative to themselves. Consequently, bad ratings tend to influence investors' beliefs – and thereby the incidence of default – more than good ratings do. Conversely, when investors have low liquidity needs, they perceive CRAs as stringent relative to themselves. Consequently, good ratings tend to influence investors' beliefs – and thereby the incidence of default – more than bad ratings do. Since, in the absence of a CRA, default is more likely when aggregate liquidity is tight, our results imply a pro-cyclical impact of CRAs: they increase default risk when it is high, and decrease default risk when it is low.

We then explore our model's implications concerning rating inflation/deflation – as measured by the frequency of good ratings over bad ones, relative to how an independent observer with the same information as the CRA would evaluate the same firms.[6] This definition con-

[4]We model debt roll-over based on the framework of Morris and Shin (2004, 2006).

[5]I.e. liquidity has no direct effect on the payoffs of the CRA, and only affects the CRA indirectly, by changing investors' behavior.

[6]Earlier studies have used different definitions of rating inflation. A number of papers define ratings as inflated when a CRA lies and gives too good ratings, while in other papers rating inflation reflects the issuer picking the best among several unbiased ratings. Our definition of rating inflation reflects the common feature

trols for the direct effect of liquidity on the incidence of default. When liquidity is tight, good ratings do not improve firms' chances of success, but bad ratings increase chances of default. Hence, because the CRA's objective is to make correct predictions, bad ratings become attractive when liquidity is tight: they allow the CRA to benefit from its own impact on the probability of default. This effect induces rating deflation. By symmetry, easy liquidity induces rating inflation. These results invoke no bias in the payoff structure of CRAs, and therefore highlight the possibility of rating inflation occurring *even without* any form of 'rating shopping' by issuers.[7]

Finally, we examine the effects of exogenous biases in CRAs' incentives. A CRA may be biased toward issuers, e.g. toward assigning good ratings due to rating shopping, or it may be biased toward bad ratings, e.g. to avoid later responsibility for failed investments. We assume that bias is common knowledge. In equilibrium, any bias is therefore taken into account by investors when interpreting ratings. If, for instance, a CRA favors issuers, investors will place little weight on good ratings, as it is then unclear if these are motivated by the outlook of rated firms or simply reflect the bias of the CRA. By contrast, a bad rating will now be given more weight, since it indicates that a CRA has observed a sufficiently negative signal to overcome its bias. We show that a bias in the incentive structure of a CRA ultimately becomes self-defeating in equilibrium. When CRAs are biased in favor of issuers, ratings on average increase the incidence of default. When CRAs are biased in favor of bad ratings, ratings on average reduce the incidence of default.

Our prediction of procyclical ratings is consistent with Griffin and Tang's (2012) finding that as the coming recession became imminent in April 2007, rating standards became markedly tougher. Moreover, our analysis may shed light on the widespread empirical find-

of an excessive frequency of positive ratings, relative to the assessment by an independent observer. See also our discussion in Section 3.3.

[7]Rating shopping refers to the ability of issuers to pay for and disclose ratings *after* they privately observe them, making it possible to choose the rating agency assigning the highest rating.

ing that ratings tend to affect investor behavior asymmetrically, with negative rating events having stronger effects than positive events, or vice versa. Our theory's empirical predictions, and relation to existing evidence, are discussed in depth in Section 4.

Theoretical research on credit rating agencies has burgeoned over the last years. Yet, Manso (2013) is the only previous paper to explore optimal CRA decisions and the feedback effects arising when ratings affect the performance of rated securities. In Manso (2013) a firm issues debt with interest payment that decreases with the firm's credit rating. Equity holders then choose the default time maximizing equity value. The CRA's objective is to set accurate ratings that inform investors about the probability of default over a given time horizon. By determining the interest rate, a rating affects the optimal default decision of the issuer, which in turn influences the rating itself. While both papers explore the feedback effects of credit ratings, our paper and Manso's therefore take substantively different approaches. In particular, while Manso's focus is on the strategic interaction between CRAs and issuers, ours on the other hand is on the strategic interaction between CRAs and creditors, allowing us to derive new results regarding the pro-cyclical nature of credit rating. Section 3.4 further highlights the starkly different normative implications of the two papers. While Manso's results advocate for a CRA bias in favor of issuers so as to reduce the incidence of default, our results imply that such a bias may have the unfortunate effect of increasing the incidence of default.

Our paper is also related to former studies of how CRAs might coordinate investors. Boot et al. (2006) develop a model with moral hazard in which credit ratings provide a focal point for firms and investors, and help select the most efficient equilibrium. By contrast, Carlson and Hale (2006) show in a global games setting similar to ours how a non-strategic CRA may induce multiple equilibria by publicly revealing its information. In global games more generally, more precise public information enhances the scope for multiplicity; see for instance Morris and Shin (2003) or Hellwig (2002). While this effect naturally arises in our model too, our focus is elsewhere. Instead, we parameterize our model to obtain a unique equilibrium. Our main innovation lies in the fact that the rating agency behaves strategically, and takes the equilibrium effects of its rating into account. Relative to the global games literature more generally, we thus contribute by studying a *strategic* sender of public information. In this respect, our paper is closer in spirit to Angeletos, Hellwig and Pavan (2006) who study the endogenous information generated by policy interventions. In a broader perspective, our paper relates to studies of self-fulfilling crises at large, such as the bank-run model of Diamond and Dybvig (1983).

An important strand of the literature explores the link between rating shopping on the one hand and rating inflation on the other. This includes, among others, Skreta and Veldkamp (2009), Sangiorgi and Spatt (2011), Bolton, Freixas and Shapiro (2012), and Opp, Opp and Harris (2013). A detailed review of this literature is given in Section 3.3. Compared to that literature, our paper highlights that rating inflation may prevail in the absence of any form of rating shopping on the part of issuers, and simply result from creditors' changing liquidity needs. Mathis et al. (2009) is also, in spirit, related to that strand of literature: A rating agency is inclined to inflate ratings in order to secure higher fees. Their focus however is on the reputational cycles resulting when CRAs first report truthfully to build up a reputation, which they then milk down by repeatedly inflating ratings.

The paper is organized as follows. The model is presented in Section 2. Section 3 solves the model and presents our results. Section 4 relates our findings to the empirical literature. Section 5 concludes. All proofs are contained in the Appendix.

2 Model

Our framework builds on Morris and Shin's (2004, 2006) model of the roll-over problem encountered by a firm or a sovereign relying on short-term debt to finance its activities. We first lay out the basic model, which we later enhance by introducing a strategic credit rating agency.

Debt roll-over. There are two time periods, $t = 1, 2$. A unit mass of investors (or creditors), indexed by i, are financing a borrower, hereafter referred to as the *firm*, using a conventional debt contract. At $t = 1$ each investor faces the option to (i) liquidate his stake in the firm for a payment normalized to 1, or (ii) roll-over his loan to the firm. In the latter case the contract specifies final-period payment V if the firm succeeds, while if the firm fails, investors receive 0.[8]

Our main goal is to examine the role played by CRAs in the context of (il)liquidity. We thus assume that the firm's ability to meet short-term claims is the sole source of uncertainty; this is the decisive factor for the firm's success or failure. Let l denote the mass of investors liquidating at time $t = 1$. The ability to meet short-term claims, and thereby avoid default, is summarized by a variable θ, unknown to creditors. One may think of θ as the firm's cash, liquid assets, or access to alternative credit lines other than the debt market. If $\theta \geq l$, the firm meets its short-term claims, and investors who in the first period chose to roll over obtain payment V at $t = 2$. If $\theta < l$, the firm defaults, and those who chose to roll over get nothing. Observe in particular that if $\theta \geq 1$, the firm survives even if all creditors were to liquidate. By contrast, default occurs with certainty if $\theta < 0$. However, if $\theta \in [0, 1)$, the firm may or may not default, depending on investors' behavior. A coordination problem then prevails: Each investor would gain if all were to roll over their loan to the firm, but no investor would gain

[8] The loan may be backed on collateral for instance, whose liquidation value is V_1 in the first period and V_2 in the second period. Following Morris and Shin (2004) our baseline model sets $V_1 = 1$ and $V_2 = 0$.

from being the only one to do so. Let I denote the indicator variable taking value 1 in case of success and 0 in case of default. Thus

$$I = 1 \Leftrightarrow \theta \geq l. \tag{1}$$

Information structure and CRA. While θ is unknown to creditors, it is common knowledge that θ is uniformly distributed over $\Delta = [\underline{\delta}, \overline{\delta}]$, where $\underline{\delta} \ll 0$, and $\overline{\delta} \gg 1$.[9] Before $t = 1$, each investor receives a private signal x_i which conditional on θ is uniformly distributed over $[\theta - \beta, \theta + \beta]$. In addition investors observe, before $t = 1$, the *rating* R of a credit rating agency. For tractability, and following much of the literature on the effect of rating agencies, e.g. Bolton et al (2012) and Mathis et al (2009), we restrict attention to a binary rating by the CRA. Thus, a rating is either good ($R = \mathbf{1}$), or bad ($R = \mathbf{0}$). We assume that the CRA receives information in the form of a private signal y, which conditional on θ is uniformly distributed over $[\theta - \alpha, \theta + \alpha]$. All signals are independent conditional on θ.

Strategies. An investor's strategy is a mapping $\sigma_i : \Delta \times \{\mathbf{0}, \mathbf{1}\} \to \{\mathsf{liquidate}, \mathsf{roll\ over}\}$ specifying whether to liquidate or roll over at time $t = 1$ as a function of the private signal x_i and the rating R.[10]

A rating strategy for the CRA on the other hand is a mapping $r : \Delta \to \{\mathbf{0}, \mathbf{1}\}$, assigning a rating to each possible signal y received by the CRA.The rating strategy r partitions Δ into $r^{-1}(\mathbf{0})$ and $r^{-1}(\mathbf{1})$.*Threshold rating strategies*, in which the CRA announces rating $\mathbf{1}$ if and only if it observes a signal y above a given threshold, play a prominent role in the analysis.[11] We let r_t denote the threshold rating strategy with threshold $t \in \Delta$. With a slight abuse of notation, we will interchangeably use R to denote the actual rating given ($\mathbf{0}$ or $\mathbf{1}$) or a subset

[9] This ensures that all relevant θ values arise with positive probability.

[10] We are slightly abusing notation here. Strictly speaking, the set of possible signals is $[\underline{\delta} - \alpha, \overline{\delta} + \alpha]$. This is however innocuous, since we are assuming $\underline{\delta} \ll 0$ and $\overline{\delta} \gg 1$.

[11] We later establish that any equilibrium must entail a threshold rating strategy.

of Δ (corresponding to $r^{-1}(\mathbf{0})$ or $r^{-1}(\mathbf{1})$). Doing so allows us to identify ratings with their actual informational content. For example, a rating from a CRA behaving according to rating strategy r_t has informational content either $y \in [\underline{\delta}, t]$, or $y \in [t, \overline{\delta}]$.

Payoffs. Investors have preferences given by

$$u(w_1, w_2) = v w_1 + w_2$$

where w_t indicates payments received in period t. The parameter v captures investors' valuation of immediate payoff relative to delayed payoff, and plays the same role as 'patience' plays in the tradition of Diamond and Dybvig (1983).[12] We will interpret v as a measure of investors' liquidity needs. Thus v is high when investors have strong immediate needs for cash, as for instance in a liquidity squeeze, or when investors face important margin calls on other positions. Importantly, v is common to all investors, and therefore reflects the aggregate state of the economy. The ratio $v/V \equiv \lambda$ plays a key role. We will say that liquidity is *tight* when λ is high, and that liquidity is *easy* when λ is low.

Following Manso (2013), we assume that the CRA is driven by a desire to preserve its track record of correct predictions.[13] There are two ways to be correct in this environment: the CRA may give a good rating to a firm that later succeeds, or it may give a bad rating to a firm that defaults. For the time being, we treat these two alternatives symmetrically, and assume that the CRA obtains a payoff normalized to 1 if it has rated **1** a successful firm or if it has rated **0** a firm which later defaults. The CRA obtains zero payoff in the two other cases, where its rating turns out to be 'incorrect'. We let $\Pi(R, I)$ denote the payoff of the

[12] See also Ennis and Keister (2000).

[13] The relevance of maintaining a track record is supported by, for instance, the following quote of Thomas McGuire, former VP at Moody's, reported in Manso (2013): 'What's driving us is primarily the issue of preserving our track record. That's our bread and butter.' Our approach is also akin to the reduced-form models of reputation used in for example Morgan and Stocken (2003) or Bolton, Freixas, and Shapiro (2012).

CRA which has given the rating R to a firm with outcome I. Hence

$$\Pi(R, I) = IR + (1 - I)(1 - R).$$

Beliefs. Before making a choice, each investor updates his beliefs regarding θ based upon the information contained in his private signal x_i and the public rating with informational content $R = [\underline{r}, \overline{r}]$. We shall say that an investor updates beliefs according to A1-A3 if he follows the following simple rules:

A1 If $x_i - \beta > \overline{r} + \alpha$, he assigns probability 1 to the event $\theta = \overline{r} + \alpha$.

A2 If $x_i + \beta < \underline{r} - \alpha$, he assigns probability 1 to the event $\theta = \underline{r} - \alpha$.

A3 If $x_i - \beta \leq \overline{r} + \alpha$ and $x_i + \beta \geq \underline{r} - \alpha$, he assumes θ is uniformly distributed on $[x - \beta, x + \beta] \cap [\underline{r} - \alpha, \overline{r} + \alpha]$.

Rules A1-A2 pin down investors' out-of-equilibrium beliefs, i.e. whenever information conveyed by the rating is inconsistent with their own information.[14] A3 approximates the Bayesian updating procedure by assuming uniform posterior distributions throughout, and plays no other role than ensuring tractability.[15]

Equilibrium. We next formulate our equilibrium concept. A profile of strategies[16] (σ^*, r^*) for investors and CRA constitute a *rating equilibrium* if and only if it satisfies the following conditions:

[14]The obvious alternative would be to set probability 1 to $\theta = x_i - \beta$ if $x_i - \beta > \sup R$, and probability 1 to $\theta = x_i + \beta$ if $x_i + \beta < \inf R$. However, that assumption opens the possibility of multiple equilibria in the investment game and thus defeats the purpose of using global games to ensure uniqueness of equilibrium. See Carlson and Van Damme (1993) and Morris and Shin (2003) for thorough discussions of the issue.

[15]Notice that rule A3 exactly coincides with Bayesian updating whenever $\underline{r} = \overline{r}$. In general however, the probability density of θ conditional on $y \in R$ tapers off at the edges of its domain. If $[\underline{r} + \alpha \leq \overline{r} - \alpha]$ for instance, then the conditional distribution is uniform on that interval and tapers off on $[\underline{r} - \alpha, \underline{r} + \alpha] \bigcup [\overline{r} - \alpha, \overline{r} + \alpha]$.

[16]We do not explicitly consider mixed strategies. Mixed strategies complicate the exposition but do not affect the results of our paper.

1. Investors update beliefs according to A1-A3.

2. Given any signal $x_i \in \Delta$ and rating $R \in \{\mathbf{0}, \mathbf{1}\}$, investors maximize expected utility: $\mathbb{E}\big[u(\sigma_i^*, \sigma_{-i}^*, r^*)\big] \geq \mathbb{E}\big[u(\sigma_i', \sigma_{-i}^*, r^*)\big]$, for all i and $\sigma_i' \neq \sigma_i^*$.

3. Given any signal $y \in \Delta$, the CRA maximizes expected payoff: $\mathbb{E}\big[\Pi(\sigma^*, r^*)\big] \geq \mathbb{E}\big[\Pi(\sigma^*, r')\big]$, for all rating strategies $r' \neq r^*$.

Besides the computational approximation of rule A3, our definition of equilibrium is thus that of a Perfect Bayesian Equilibrium.

Finally, we impose a number of parameter restrictions. Observe that if $\lambda \geq 1$, then all investors always liquidate. If on the other hand $\lambda = 0$, then all investors always roll over. So $\lambda \in (0, 1)$ is the critical region of interest, which we henceforth restrict attention to. We assume throughout that α and β are strictly greater than 1/2. Furthermore, overly precise information on the part of the CRA re-introduces multiple equilibria into the Diamond and Dybvig (1983) framework of self-fulfilling crises, thus defeating the purpose of using global games to retrieve equilibrium uniqueness.[17] In our model, if $\alpha < \beta$, the rating of the CRA is then sufficiently informative to induce multiple equilibria. We thus assume $\alpha > \beta$, in order to preserve equilibrium uniqueness. Finally, to guarantee the existence of an equilibrium, we assume that $1/(2\beta + 1) < \lambda < 1 - 1/(2\beta + 1)$.

3 Analysis and results

The equilibrium is solved backwards, i.e. by first investigating the impact of a given rating on the incidence of default. We define for that purpose the *investment game* given rating R as the game played among investors observing R and following rules A1-A3. Section 3.1 studies that game. While the investment game treats ratings as exogenous, section 3.2 endogenizes

[17]See e.g. Morris and Shin (2003), or Proposition 2 in Carlson and Hale (2006).

ratings by incorporating strategic behavior on the part of the credit rating agency. This section addresses the central question of our paper: do credit ratings affect the incidence of default and, if so, how? Sections 3.3 examines the implications of our analysis concerning rating inflation/deflation, while section 3.4 extends the basic model in order to explore the effect of possible biases on the part of credit rating agencies.

3.1 The investment game

Equilibrium in the investment game is characterized by two threshold values.[18] There is first a threshold value for investors' signals, denoted $x^*(R)$, such that investors observing $x_i < x^*(R)$ liquidate while investors observing $x_i \geq x^*(R)$ roll over. There is then a threshold value for θ, denoted $\theta^*(R)$, below which the firm defaults and above which it succeeds. To shorten notation, we omit the dependence of the thresholds on the rating R where this is unlikely to create confusion.

We will say that an investor is a *marginal investor* if he receives the private signal $x_i = x^*(R)$. A marginal investor is thus indifferent between liquidating and rolling over. The indifference equation of the marginal investor can be written as

$$\mathbb{P}(\theta > \theta^* | x_i = x^*, R) = \frac{v}{V} = \lambda. \tag{2}$$

Since all investors are ex ante identical, then given θ, the mass l of investors liquidating equals the probability that an arbitrary investor receives signal $x_i < x^*$. Thus

$$l = \mathbb{P}(x_i < x^* | \theta). \tag{3}$$

[18]The analysis of the investment game is standard in the *global games* literature. See e.g. Morris and Shin (1998) or Carlson and Hale (2006).

Combining equations (1) and (3), θ^* is given by

$$\theta^* = \mathbb{P}(x_i < x^* | \theta = \theta^*). \tag{4}$$

To set a benchmark, we will start by solving the investment game given rating $R = \Delta$. In this case, the rating provides no information, thus capturing the situation prevailing in the absence of a CRA. When $R = \Delta$, the posterior beliefs of an investor observing signal x_i are uniform on $[x_i - \beta, x_i + \beta]$. Equation (2) thus yields

$$x^* = \theta^* - \beta + 2\beta\lambda \tag{5}$$

while equation (4) gives

$$\theta^* = \frac{x^* + \beta}{2\beta + 1} \tag{6}$$

Finally, (5) and (6) together yield

$$\theta^* = \lambda \tag{7}$$

$$x^* = \lambda(2\beta + 1) - \beta \tag{8}$$

Equation (7) establishes the key link existing in our model – independently of the CRA – between liquidity on the one hand and default risk on the other: the easier liquidity (i.e. the lower λ), the lower the chance of default (i.e. the lower θ^*).

We next turn to the analysis of the investment game given a rating $R = [\underline{r}, \overline{r}] \neq \Delta$. While ratings have no impact on equation (4), they affect investors' beliefs and the indifference equation of the marginal investor, yielding

$$\mathbb{P}(\theta > \theta^* | \theta \in [x^* - \beta, x^* + \beta] \cap [\underline{r} - \alpha, \overline{r} + \alpha]) = \lambda \tag{9}$$

If the interval $[\underline{r} - \alpha, \overline{r} + \alpha]$ covers the support of the marginal investor's beliefs, which from (8) is given by $[\lambda(2\beta+1) - 2\beta, \lambda(2\beta+1)]$, then the marginal investor's behavior is unchanged by the rating. The equilibrium is therefore unaffected. If, however, the rating R induces the marginal investor to revise his beliefs concerning θ, then the rating will affect his behavior and, thereby, both threshold equilibrium values. A 'negative' rating such that the upper bound of the rating plus noise lies below the upper bound of the marginal investor's beliefs, i.e. such that $\overline{r} + \alpha < \lambda(2\beta+1)$, induces him to liquidate. The threshold x^* thus rises, causing in turn θ^* to rise and making default more likely. In a similar way, a rather 'positive' rating such that the lower bound of the rating minus noise lies above the lower bound of the marginal investor's beliefs, i.e. such that $\underline{r} - \alpha > \lambda(2\beta + 1) - 2\beta$, induces him to strictly prefer rolling over; x^* decreases, which in turn causes θ^* to fall, making default less likely. Lastly, whenever $\theta < 1$ a sufficiently negative rating affects all investors and triggers default. Similarly, whenever $\theta \geq 0$ a sufficiently positive rating affects all investors and triggers success. The following Lemma summarizes these observations.

Lemma 1 *An equilibrium of the* investment game given rating R *exists, and is unique. This equilibrium is characterized by the threshold* $\theta^*(R) \in [0, 1]$ *such that:* $I = 1 \Leftrightarrow \theta \geq \theta^*(R)$.

In particular, ratings affect θ^* *in the following way:*

1. $\theta^*(\Delta) = \lambda$
2. $\overline{r} + \alpha < (2\beta + 1)\lambda \;\Leftrightarrow\; \theta^*(R) > \lambda$
3. $\underline{r} - \alpha > (2\beta + 1)\lambda - 2\beta \;\Leftrightarrow\; \theta^*(R) < \lambda$
4. $\overline{r} + \alpha < 1 \Leftrightarrow \theta^*(R) = 1$
5. $\underline{r} - \alpha > 0 \Leftrightarrow \theta^*(R) = 0$

3.2 The equilibrium impact of credit ratings

This section endogenizes ratings by incorporating strategic behavior on the part of the credit rating agency. We show that rating equilibria exist, and explore their fundamental properties.

Our framework possesses a trivial equilibrium, henceforth denoted (σ_0, r_0), in which the CRA assigns ratings independently of the signal it receives.[19] Since it provides no additional information to creditors, this *babbling* equilibrium captures the situation prevailing in the absence of a CRA and, as such, provides the natural benchmark from which to draw comparisons. We begin this section with the necessary notation and definitions.

Let $\mathbb{P}(I = 1|R, y)$ denote the probability of success as seen from the perspective of a CRA observing signal y and announcing rating R; thus $\mathbb{P}(I = 1|R, y) = \mathbb{P}\big(\theta \geq \theta^*(R)|y\big)$. Given rating strategy r, define the function $Q^r : \Delta \to [0, 1]$ such that

$$Q^r(y) = \mathbb{P}(I = 1|r(y), y).$$

$Q^r(.)$ thus records the success probability as a function of the signal y, given rating strategy r.

The difference $Q^r - Q^{r_0}$ provides a natural measure of the impact of rating strategy r on the incidence of default. A CRA following rating strategy r where $Q^r = Q^{r_0}$, in particular, leaves the firm's probability of default unaffected. As such, a rating strategy $r \neq r_0$ but satisfying $Q^r = Q^{r_0}$ will be called *reducible* to r_0. By extension, a rating equilibrium (σ, r) will be called irreducible if and only if r cannot be reduced to r_0.

Let $Q^r > Q^{r_0}$ denote the case where $Q^r(y) \geq Q^{r_0}(y)$ for all $y \in \Delta$, with strict inequality for some $y \in \Delta$. A rating strategy r such that $Q^r > Q^{r_0}$ therefore unambiguously reduces the

[19] Since for expositional purposes we are restricting attention to pure strategies, r_0 here denotes any rating strategy which assigns one and the same rating to all signals received by the CRA.

incidence of default. By symmetry, let $Q^r < Q^{r_0}$ denote the case where the rating strategy r increases the incidence of default. Note that default induced by coordination failure from investors is inefficient. Thus, a rating strategy which reduces (respectively, increases) the incidence of default affects welfare positively (respectively, negatively).[20]

Finally, observe that if a threshold rating strategy r_t reduces the incidence of default, so that $Q^{r_t} > Q^{r_0}$, then $Q^{r_t}(y)$ must equal $Q^{r_0}(y)$ for all $y < t$. This property follows from the fact that, as far as threshold rating strategies are concerned, a bad rating can never be of any help to the firm. In a similar way, a good rating cannot be detrimental to the firm. Hence, if $Q^{r_t} < Q^{r_0}$ then $Q^{r_t}(y)$ must equal $Q^{r_0}(y)$ for all $y > t$.

We begin the analysis with a simple observation concerning the CRA's equilibrium behavior. For any rating R, note that $\mathbb{P}(I = 1|R, y)$ is non-decreasing in y. An equilibrium rating strategy must therefore be a threshold rating strategy. We thus restrict attention to this class of strategies. To shorten notation, let in what follows $R_t^- = [\underline{\delta}, t]$, and $R_t^+ = [t, \overline{\delta}]$. A CRA using strategy r_t thus communicates that $y \in R_t^+$ if its rating is good, and that $y \in R_t^-$ if its rating is bad.

A threshold rating strategy r_t is part of a rating equilibrium if and only if a CRA observing $y = t$ is indifferent between announcing a good rating and announcing a bad rating. Thus, in equilibrium, we have

$$\mathbb{P}(I = 1|R_t^+, t) = \mathbb{P}(I = 0|R_t^-, t). \tag{10}$$

The left-hand side of this equation is the probability that the firm succeeds, as seen from the perspective of a CRA observing signal $y = t$ and announcing $R = \mathbf{1}$. The right-hand side is the probability that a firm defaults, as seen from the perspective of a CRA observing $y = t$ and announcing $R = \mathbf{0}$.

[20] See Lemma 2 in the Appendix.

In order to understand the equilibrium mechanics, it is useful to start from the special case where $\lambda = 1/2$. When $\lambda = 1/2$, a CRA using threshold $t = 1/2$ never affects investor behavior, irrespective of whether it announces $R = \mathbf{1}$ or $R = \mathbf{0}$ (Lemma 1); in both cases $\theta^* = x^* = 1/2$. If it receives signal $y = 1/2$, the CRA assigns 50% chance to the firm's success and 50% chance to the firm's default, so that (10) holds. For future reference, note that in this case $t = x^*$, which means that CRA and investors base their decision on the same threshold.[21]

Deviations from $\lambda = 1/2$ on the other hand give rise to a structural wedge between the CRA and investors: for given success probabilities, λ affects investors' expected payoffs, but leaves CRA payoffs unaffected. As λ increases, investors require a higher probability of success in order to roll over their loans. In particular, they start demanding greater success probabilities than what a CRA requires in order to announce $R = \mathbf{1}$. A marginal investor must therefore observe $x^* > t = \lambda$. In turn, as x^* rises above t, bad ratings tend to become more informative to a marginal investor (while good ratings tend to become less informative). At $\lambda = \alpha/2\beta > 1/2$ (c.f. Lemma 1), a bad rating starts affecting outcomes, raising θ^* above λ. For λ high, therefore, ratings on average increase the incidence of default. But high values of λ are associated with high default risk (Lemma 1). CRAs thus exhibit a pro-cyclical impact: they increase default risk when it is high, and decrease it when it is low. The next proposition summarizes our findings.

Proposition 1 *A rating equilibrium always exists. If* $1 - \frac{\alpha}{2\beta} < \lambda < \frac{\alpha}{2\beta}$, *then the babbling equilibrium is the only irreducible rating equilibrium. There exists otherwise exactly one other irreducible rating equilibrium,* (σ^*, r_{t^*}). *In this equilibrium, credit ratings have a pro-cyclical impact:*

1. *If* $\lambda < 1 - \frac{\alpha}{2\beta}$ *then* $Q^{r_{t^*}} > Q^{r_0}$.

2. *If* $\lambda > \frac{\alpha}{2\beta}$ *then* $Q^{r_{t^*}} < Q^{r_0}$.

[21] Note also that the equilibrium described here is in fact reducible to r_0.

Tight liquidity motivates restraint from investors, but is of no direct concern to CRAs. When liquidity is tight, investors thus perceive CRAs as being lenient relative to themselves. Consequently, bad ratings tend to influence investors' beliefs – and thereby the incidence of default – more than good ratings do. Conversely, when investors have low liquidity needs, they perceive CRAs as stringent relative to themselves. Consequently, good ratings tend to influence investors' beliefs – and thereby the incidence of default – more than bad ratings do.

Figure 1 illustrates the effect of credit ratings. In the upper panel, $\lambda > \alpha/2\beta$ and marginal investors' private signal x^* lies above the equilibrium threshold t^*. Good ratings, corresponding to $y > t^*$, thus have no impact. Bad ratings on the other hand, corresponding to $y < t^*$, move θ^* up – potentially affecting outcomes and therefore lowering $Q^{r_{t^*}}$ below Q^{r_0}. At $y = \lambda - \alpha$ default occurs with probability 1 irrespective of the rating. $Q^{r_{t^*}}$ and Q^{r_0} thus again coincide for $y \leq \lambda - \alpha$, despite the rating's adverse effect on θ^*. The lower panel displays effect of credit rating where $\lambda < 1 - \frac{\alpha}{2\beta}$, so that bad ratings have no effect while good ratings move θ^* down.

3.3 Rating inflation

The phenomenon of rating inflation refers to a CRA's excessive tendency to give positive ratings. We measure in this paper rating inflation by the relative frequency of good ratings over bad ones, relative to the assessment by an independent observer with the same information as the CRA.

By Lemma 1, in the absence of a CRA, $\theta^* = \lambda$. Hence, whether an independent observer interprets his signal as 'good news' depends on whether this signal is above or below λ. A CRA using strategy r_t on the other hand announces a good rating for any signal above t and a bad rating for any signal below t. Thus, if $t < \lambda$, the frequency of good ratings is higher than the frequency with which an independent observer receives signals he interprets as 'good news'.

(a) Tight Liquidity

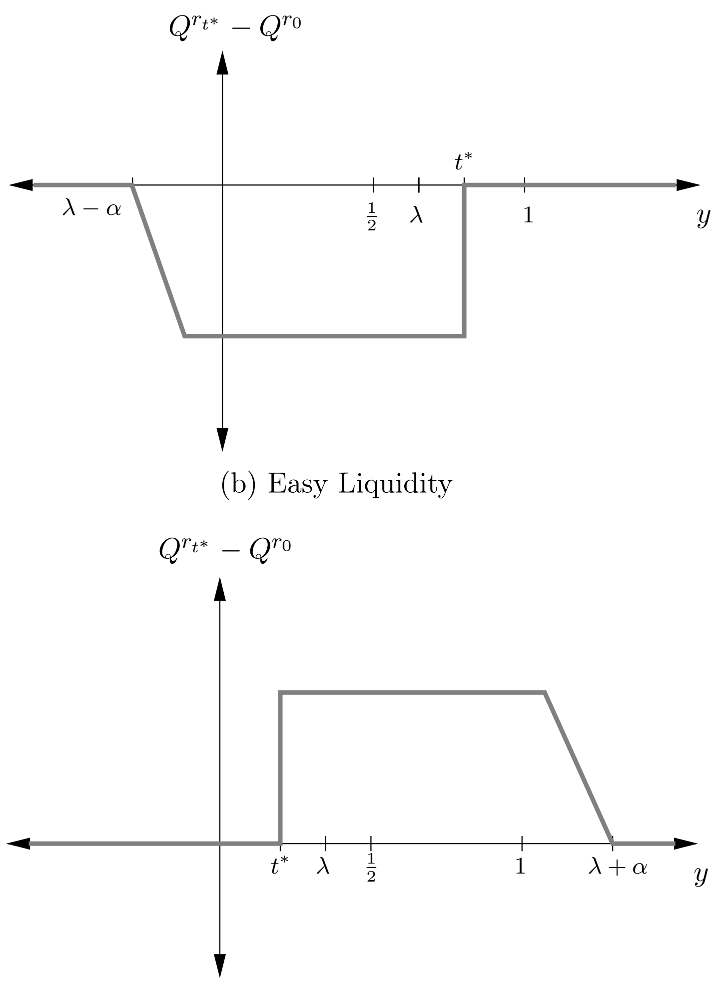

Figure 1: Liquidity and the Impact of Credit Rating.

Similarly, if $t > \lambda$, the frequency of bad ratings is higher than the frequency with which an independent observer receives signals he interprets as 'bad news'. These observations motivate the following definitions:[22]

Definition 1 *A threshold rating strategy r_t exhibits* rating inflation *if $t < \lambda$, and exhibits* rating deflation *if $t > \lambda$.*

Given rating equilibrium (σ^*, r_{t^*}), we also define

$$\pi(\sigma^*, r_{t^*}) = \mathbb{E}(IR + (1-I)(1-R)|\sigma^*, r_{t^*})$$

recording the sum of probabilities that (i) the CRA announces $R = \mathbf{1}$ and the firm succeeds, and (ii) the CRA announces $R = \mathbf{0}$ and the firm defaults. The quantity π thus provides a useful measure of ratings' *predictive power* within the equilibrium (σ^*, r_{t^*}). For comparison, $\pi_0 = \mathbb{P}(\theta > \theta^*(\Delta)|y > \theta^*(\Delta)) + \mathbb{P}(\theta < \theta^*(\Delta)|y < \theta^*(\Delta))$ measures the predictive power of an independent observer receiving an unbiased signal of θ in the absence of a CRA.

We next show that in equilibrium our model exhibits rating inflation when λ is low and rating deflation when it is high. When $\lambda > \alpha/2\beta$, a bad rating induces an upward – and thus adverse – shift of θ^* (Proposition 1). Thus, in that region, the indifference equation (10) no longer holds for $t = \lambda$, and a CRA receiving signal $y = \lambda$ finds it in its best interest to announce $R = \mathbf{0}$: doing so allows it to benefit from its own impact on the probability of default. This implies that in equilibrium $t > \lambda$: ratings exhibit deflation. Note furthermore that in this case, in equilibrium both sides of equation (10) exhibit probabilities greater than 50%.

[22] In our model, a worsening of creditors' liquidity needs (i.e. a rise in λ) mechanically raises the incidence of default, and renders bad ratings attractive from CRAs' viewpoint – irrespective of any perverse incentives these might have. Notice that our definition of rating inflation controls for this direct effect of λ on the incidence of default.

Deviations from $\lambda = 1/2$ thus (i) create rating inflation/deflation and (ii) increase ratings' predictive power.

Proposition 2 *The unique irreducible rating equilibrium, (σ^*, r_{t^*}), exhibits rating inflation if $\lambda < 1 - \frac{\alpha}{2\beta}$ and rating deflation if $\lambda > \frac{\alpha}{2\beta}$. Moreover, deviations from $\lambda = 1/2$ increase ratings' predictive power: $\pi(\sigma^*, r_{t^*}) > \pi_0$ whenever $\lambda < 1 - \frac{\alpha}{2\beta}$ or $\lambda > \frac{\alpha}{2\beta}$.*

Rating inflation has recently attracted much attention. The theoretical literature has focused on the role of *rating shopping* – the ability of issuers to pay for and disclose ratings *after* they privately observe them – in generating rating inflation. In Skreta and Veldkamp (2009) a series of non-strategic CRAs truthfully report a noisy signal of an asset's return. Issuers then exploit investors' failure to account for the fact that they strategically select the best of all ratings. Published ratings are thus biased signals of assets' true values, and prices distorted. Sangiorgi and Spatt (2011) extend the analysis by showing that similar outcomes result when investors behave rationally but are uncertain about the number of ratings observed by issuers. In Bolton et al. (2012), an issuer can at a fixed price increase the quantity sold to unsophisticated investors who take ratings at face value. Good ratings thus allow issuers to raise profits, creating high willingness to pay for these ratings. This in turn incentivizes CRAs to produce good ratings, for which they receive higher fees. Opp et al. (2013) build on this approach and show that introducing rational investors dampens rating inflation, since inflated ratings are less informative – and therefore less valuable – for investors.

Our paper in no way contends the role of rating shopping or distorted incentives in generating rating inflation (see also Section 3.4 below concerning biased CRAs). Instead, we highlight that rating inflation may prevail *even without* any form of rating shopping on the part of issuers. Because a CRA may affect the outcome by its choice of rating, it may deliberately choose to give a different rating than an independent observer would do, in order to increase the probability of a correct prediction. Note that in contrast to earlier studies, in our paper there is rating inflation even though published ratings are 'truthful', in the sense that CRAs do not lie and issuers do not select the best among alternative ratings. Furthermore, in our model rating inflation or deflation depends on creditors' changing liquidity needs. When creditors' have high liquidity needs, good ratings do not improve firms' chances of success, but bad ratings increase chances of default. Thus, the CRA favor bad ratings over good ones, inducing rating deflation, to increase the probability of a correct prediction. Similarly, when liquidity is abundant and investors hungry for yield, bad ratings do not worsen a firm's prospects, while good ratings improve chances of success. CRAs then favor good ratings over bad ones, inducing rating inflation. To the best of our knowledge, these insights are novel.

3.4 Biased Credit Rating Agencies

In this section, we examine the consequences of exogenously biased incentives on the part of the CRA.

While rating shopping biases CRAs in favor of issuers, the bias could in general go both ways. In the case of unsolicited rating for instance – such as in the case of sovereign debt – CRAs may be more afraid of mistakenly guiding investors toward under-performing assets than of mistakenly guiding them away from successful ones. Moreover, the design of appropriate incentive structures in the ratings industry is a potentially important policy instrument for regulators. These considerations motivate a general examination of the effect of biased incentives of the CRA.

We modify our specification of CRA payoffs to:

$$\Pi(R, I) = \rho_1 IR + \rho_0(1 - I)(1 - R) \tag{11}$$

A CRA with $\rho_1 > \rho_0$ is biased in favor of issuers, while $\rho_1 < \rho_0$ implies a bias toward bad ratings. This is intended as a parsimonious way to represent biases in any direction, without taking a stance on the fundamental cause of them.

Biased incentives have the expected effect on rating standards, in the sense that a bias in favor of issuers leads to rating inflation, while the opposite bias leads to rating deflation. The result that an issuer bias leads to rating inflation is in line with the rating-shopping literature discussed in the previous section. The next proposition shows, however, that an issuer bias need not reduce the incidence of default; quite on the contrary.

Proposition 3 *Consider CRA payoffs given in (11). A rating equilibrium always exists. If* $\frac{\frac{\beta}{\alpha}\lambda}{1-\frac{\beta}{\alpha}\lambda} < \frac{\rho_0}{\rho_1} < \frac{1-\frac{\beta}{\alpha}(1-\lambda)}{\frac{\beta}{\alpha}(1-\lambda)}$*, the babbling equilibrium is the only irreducible rating equilibrium. There exists otherwise exactly one other irreducible rating equilibrium,* (σ^*, r_{t^*})*. In this equilibrium, any bias on the part of the CRA is 'self-defeating':*

1. *if* $\frac{\rho_0}{\rho_1} > \frac{1-\frac{\beta}{\alpha}(1-\lambda)}{\frac{\beta}{\alpha}(1-\lambda)}$*, then* $Q^{r_{t^*}} > Q^{r_0}$*.*
2. *if* $\frac{\rho_0}{\rho_1} < \frac{\frac{\beta}{\alpha}\lambda}{1-\frac{\beta}{\alpha}\lambda}$*, then* $Q^{r_{t^*}} < Q^{r_0}$*.*

Any bias in the incentive structure of a CRA ultimately becomes self-defeating in equilibrium. When the fact that CRAs are biased in favor of issuers is common knowledge, ratings on average increase the incidence of default. When CRAs are biased toward bad ratings, ratings on average reduce the incidence of default.

The underlying mechanism is the following. Investors know CRAs' systematic bias, and take this into account when interpreting ratings. Hence, if CRAs favor issuers, investors place little weight on good ratings, as it is then unclear if these are motivated by the outlook of rated firms or simply reflect CRA bias. By contrast, a bad rating is now given more weight, since it indicates that a CRA has observed a sufficiently negative signal to overcome its bias. In short, rational investors discount ratings according to the known biases of CRAs.

The previous insight has important practical implications. An issuer bias dilutes the impact of good ratings. Thus, the objective of preventing inefficient default is best achieved by means of a cautious bias on the part of CRAs. This view contrasts sharply with the arguments of, for example, Manso (2013), in support of the issuer-pay model – on the grounds that good ratings potentially limit the incidence of default. Fundamentally, the reason why our conclusion differs, is that Manso takes investors' responses to ratings as given mechanically while we study investor behavior as an equilibrium outcome in which CRA biases are rationally taken into account.

4 Empirical implications

We here discuss how our findings relate to the existing empirical literature on credit ratings, as well as testable predictions for future empirical research.

Proposition 1 highlights the pro-cyclical impact of credit ratings. This finding supports the views held by several observers in the context of the Eurozone crisis, as typified by the joint statement of Merkel and Sarkozy given in the introduction.[23] Empirical research on the issue is unfortunately scant, and existing evidence remains inconclusive. In a rare study of rating cyclicality, Amato and Furfine (2004) find that while average ratings are acyclical, new ratings and rating changes typically have a procyclical effect. Related in spirit, Ferri, Liu and Stiglitz (1999) have argued that during the East Asian crisis, CRAs attached 'higher weights to their qualitative judgement than to the economic fundamentals [...], such behaviour may have helped to exacerbate the boom and bust cycle in East Asia.' Reisen and von Maltzan (1999), on the other hand, suggest a more neutral effect from CRAs. The prediction that credit ratings have a procyclical impact is testable, and should be scrutinized by future empirical research.

[23] Another example is Helmut Reisen (2010) Head of Research, OECD Development Centre: 'Unless sovereign ratings can be turned into proper early warning systems, they will continue to add to the instability of international capital flows, to make returns to investors more volatile than they need be, and to reduce the benefits of capital markets for recipient countries.'

Proposition 2 highlights that rating standards tend to be procyclical, with rating inflation in boom times and rating deflation during busts. This prediction is consistent with Griffin and Tang (2012) who closely study the practice of a top credit rating agency. They find that from 2003 to 2007, this CRA frequently made upward adjustments to the its models' assessments of rated objects, and that the extent of the adjustments increased substantially over that period. As one might expect, assets with larger initial adjustments experienced more severe downgrading later on. Moreover, rating standards suddenly became more stringent in April 2007, a time when the forthcoming recession started to become visible. Ashcraft, Goldsmith-Pinkham and Vickery (2009) also point toward cyclical rating standards. They find a progressive decline in rating standards around the MBS market peak between the start of 2005 and mid-2007, i.e. prior to the financial crisis, when credit was cheap and in abundant supply.

An intriguing pattern, that has been particularly well-documented by the empirical literature, is that credit ratings tend to have an asymmetric effect. The pattern was first emphasized by Holthausen and Leftwich (1986), while later studies pointing toward asymmetries include, amongst many others, Galil and Soffer (2011), Afonso, Furceri and Gomes (2012), Hull et. al (2004), Norden and Weber (2004). The common finding here is that adverse rating events tend to shift prices, whereas the effects of favorable rating events are weak or non-existent. Our findings might shed light on this asymmetry.[24] In particular, for a given state of the market, Proposition 3 implies that if – as is widely believed[25] – CRAs are biased in favor of issuers, then bad ratings matter more than good ones. Likewise, Proposition 2 implies that even if CRAs are unbiased, the same asymmetry occurs if markets conditions are weak. The study

[24]Given that our model is static, we cannot strictly account for downgrades or upgrades. However, if a CRA has given a rating in the past, then as time elapses this rating looses relevance. By the time a new rating is announced, the situation may therefore to some extent be similar to one where no previous rating existed, as our model assumes.

[25]See e.g. Pagano and Volpin, 2010.

by Ismailescu and Kazemi (2010) is one of the few papers finding evidence of the opposite asymmetry. They consider sovereign debt of emerging economies. In light of our model, one can hypothesize that their findings are driven by a negative CRA bias for this specific asset class.

More generally, our paper implies that incentives and underlying market conditions will determine both the impact and standards of credit ratings. These relationships are in principle testable, and will hopefully be addressed by future empirical work in the field.

5 Conclusion

Credit rating agencies have been criticized on different and often opposing grounds, in particular for being too lenient before the financial crisis, and for contributing to the downturn after the crisis. We explore these issues by developing an equilibrium model of credit rating. We find that when aggregate liquidity is easy, ratings are inflated and on average decrease the incidence of default. By contrast, when liquidity is tight, ratings are deflated and on average increase the incidence of default. While existing studies focus on how an issuer bias in CRA payoffs generates rating inflation, our study is the first to show how rating inflation may arise due to booming market conditions. Moreover, we show that biased incentives on the part of CRAs are ultimately self-defeating. To the best of our knowledge, our paper is the first to simultaneously: (i) account for strategic behavior on the part of CRAs, (ii) allow for the possibility that ratings affect the performance of the rated objects, and (iii) endogenize investors' response to credit ratings.

Our paper contributes to the ongoing debate on how CRAs should be paid. A widely held concern is that existing remuneration schemes, in which CRAs typically are paid by issuers, cause too generous ratings.[26] To resolve this issue, one suggestion has been that investors –

[26]For an overview of the debate, see e.g. Mathis, McAndrews and Rochet (2009) and Pagano and Volpin

not issuers – should finance CRAs. Our analysis provides support for this view, but from a different angle than the arguments that have been used so far: Credit ratings can serve as a welfare improving coordination device, but only if agencies have a sufficiently negatively biased incentive structure. If rating agencies are known to side with issuers, good ratings will be discounted by investors while bad ratings may have strong effects on investors' beliefs. This view stands in sharp contrast with the arguments developed by, for example, Manso (2013), in support of the issuer-pay model – on the grounds that good ratings potentially limit the incidence of default. Fundamentally, the reason why our conclusion differs, is that Manso treats investors' responses to credit ratings as exogenous, while we study investor behavior as an equilibrium outcome in which CRA biases are rationally taken into account. The role played by institutionally constrained investors – who are forced to sell once assets drop below investment grade – is thus crucial to this issue. Recent policy initiatives and reforms in the U.S. and E.U. explicitly aim to reduce rating reliance in law and regulation.[27] These developments will give added weight to our conclusions.

Appendix

Proof of Lemma 1: Say that an equilibrium of the investment game is interior if in this equilibrium an investor's behavior is contingent on his private signal x. Otherwise say that the equilibrium is a corner equilibrium.

As indicated in the body of the paper, any interior equilibrium is characterized by a pair (x^*, θ^*) satisfying equations (4) and (9), and repeated here:

(2010).

[27] In the U.S., the 2010 Dodd Frank Act removes statutory references to credit rating agencies, and calls for federal regulators to review and modify existing regulations to avoid relying on credit ratings as the sole assessment of creditworthiness (U.S. Securities and Exchange Commission, 2014). Similar initiatives have been taken in the EU (European Commission, 2013).

$$\begin{cases} \theta^* = \mathbb{P}(x < x^* | \theta = \theta^*) \\ \mathbb{P}(\theta > \theta^* | \theta \in [x^* - \beta, x^* + \beta] \cap [\underline{r} - \alpha, \overline{r} + \alpha]) = \lambda \end{cases} \quad (12)$$

Note in particular that the second equation of the system defines a broken line in the (x^*, θ^*)-plane joining A, B, C, D where

$$A = (\underline{r} - \alpha - \beta, \underline{r} - \alpha)$$

$$B = (\underline{r} - \alpha + \beta, \underline{r} - \alpha + 2\beta(1 - \lambda))$$

$$C = (\overline{r} + \alpha - \beta, \overline{r} + \alpha - 2\beta\lambda)$$

$$D = (\overline{r} + \alpha + \beta, \overline{r} + \alpha)$$

By (12) an interior equilibrium thus exists if and only if (i) $\overline{r} + \alpha > 1$, and (ii) $\underline{r} - \alpha < 0$. If $\overline{r} + \alpha \leq 1$, a corner equilibrium exists in which all investors liquidate, irrespective of private signals. Similarly, if $\underline{r} - \alpha \geq 0$ then a corner equilibrium exists in which all investors roll over, irrespective of private signals. Hence an equilibrium always exists. Furthermore the equilibrium is unique since any time (i) and (ii) hold corner equilibria are precluded (investors receiving private signal above 1 must roll over while investors receiving private signal below 0 must liquidate).

Properties 1-5 follow from simple computations using (12).

■

Lemma 2 *A rating equilibrium induces higher (lower) welfare than the babbling equilibrium if it induces lower (higher) probability of default than the latter. Let (σ^*, r_{t^*}) denote a rating equilibrium; formally:*

1. $Q^{r_{t^*}} > Q^{r_0} \Rightarrow \mathbb{E}[u(\sigma^*, r_{t^*})] > \mathbb{E}[u(\sigma_0, r_0)]$

2. $Q^{r_{t^*}} < Q^{r_0} \Rightarrow \mathbb{E}[u(\sigma^*, r_{t^*})] < \mathbb{E}[u(\sigma_0, r_0)]$

Proof: We show the proof of the second part. The proof of the first part is similar. Observe that $Q^{r_{t^*}} < Q^{r_0}$ implies $x^*(R^-_{t^*}) > x^*(\Delta) = x^*(R^+_{t^*})$. By definition of x^*, in equilibrium an investor observing signal $x_i < x^*$ has utility v while an investor observing signal $x_i > x^*$ has (expected) utility strictly more than v. Averaging ex ante (i.e. before the realization of θ) thus gives the desired result.

■

Proof of Proposition 1: Rewriting (10), the threshold rating strategy r_t is part of a rating equilibrium if and only if t satisfies

$$\mathbb{P}(I = 1|R^+_t, t) = 1 - \mathbb{P}(I = 1|R^-_t, t). \tag{13}$$

By Lemma 1 the LHS of the equation is 1 for $t - \alpha > 0$ and 0 for $t + \alpha < \lambda$, while the RHS of the equation is 1 for $t + \alpha < 1$ and 0 for $t - \alpha > \lambda$. Moreover, the LHS of the equation is strictly increasing for $t \in [\lambda - \alpha, \alpha]$ and the RHS of the equation strictly decreasing for $t \in [1 - \alpha, \lambda + \alpha]$. The two curves thus cross exactly once. Let t^* denote this unique t.

We next explore the conditions under which r_{t^*} is irreducible. By Lemma 1, a necessary condition is that one of the following two condition holds (note that at most one of the two conditions holds at once):

Condition 1: $t^* + \alpha < (2\beta + 1)\lambda$ (in which case $\theta^*(R^-_{t^*}) > \lambda$).

Condition 2: $t^* - \alpha > (2\beta + 1)\lambda - 2\beta$ (in which case $\theta^*(R^+_{t^*}) < \lambda$).

We next show that if either condition holds then $Q^{r_{t^*}} \neq Q^{r_0}$, i.e. r_{t^*} is irreducible. By (13), note that

$$t^* = \frac{\theta^*(R_{t^*}^-) + \theta^*(R_{t^*}^+)}{2}. \tag{14}$$

Thus, in particular if (say) condition 1 holds then $t^* > \lambda$ (recall that in that case $\theta^*(R_{t^*}^-) > \lambda = \theta^*(R_{t^*}^+)$). Suppose now for a contradiction that $Q^{r_{t^*}} = Q^{r_0}$. Since $\theta^*(R_{t^*}^-) > \lambda$ it must then be that $t^* + \alpha < \lambda$, i.e. default occurs with certainty from the point of view of the CRA observing t^*. Be we then have $t^* + \alpha < \lambda$ and $t^* > \lambda$, which is the desired contradiction. This finishes to show that if either of conditions 1-2 holds then r_{t^*} is irreducible.

The proof is concluded by noting that condition 1 is equivalent to

$$\mathbb{P}(I = 1 | R_t^+, y = t = (2\beta + 1)\lambda - \alpha) > \big(1 - \mathbb{P}(I = 1 | R_t^-, y = t = (2\beta + 1)\lambda - \alpha)\big) \tag{15}$$

while condition 2 is equivalent to

$$\mathbb{P}(I = 1 | R_t^+, y = t = (2\beta+1)\lambda - 2\beta + \alpha) < \big(1 - \mathbb{P}(I = 1 | R_t^-, y = t = (2\beta+1)\lambda - 2\beta + \alpha)\big). \tag{16}$$

Substituting in (15) using Lemma 1 gives

$$\frac{[(2\beta + 1)\lambda - \alpha] + \alpha - \lambda}{2\alpha} > 1 - \frac{[(2\beta + 1)\lambda - \alpha] + \alpha - \lambda}{2\alpha}$$

which simplifies to $2\beta\lambda > \alpha$.

Substituting in (16) using Lemma 1 gives

$$\frac{[(2\beta + 1)\lambda - 2\beta + \alpha] + \alpha - \lambda}{2\alpha} < 1 - \frac{[(2\beta + 1)\lambda - 2\beta + \alpha] + \alpha - \lambda}{2\alpha}$$

which simplifies to $2\beta\lambda < 2\beta - \alpha$.

■

Proof of Proposition 2: The first part follows from (14) and the observation that $\theta^*(R^-_{t^*}) > \lambda = \theta^*(R^+_{t^*})$ if condition 1 holds, while $\theta^*(R^-_{t^*}) = \lambda > \theta^*(R^+_{t^*})$ if condition 2 holds.

That ratings' predictive power increases in both cases follows from the facts that (i) $\mathbb{P}(I = 1|R^+_{t^*}, t^*) = \mathbb{P}(I = 0|R^-_{t^*}, t^*)$, (ii) $\mathbb{P}(I = 1|R^+_{t^*}, t^*) > \mathbb{P}(I = 1|R^-_{t^*}, t^*)$, and (iii) $\mathbb{P}(I = 1|R^+_{t^*}, t^*) + \mathbb{P}(I = 0|R^+_{t^*}, t^*) = 1$. These together imply $\mathbb{P}(I = 1|R^+_{t^*}, t^*) = \mathbb{P}(I = 0|R^-_{t^*}, t^*) > 1/2$.

■

Proof of Proposition 3: Follows the steps of the proof of Proposition 1, substituting (13) with

$$\rho_1 \mathbb{P}(I = 1|R^+_t, t) = \rho_0\big(1 - \mathbb{P}(I = 1|R^-_t, t)\big). \tag{17}$$

■

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