# A Theory of Repurchase Agreements, Collateral Re-use, and Repo Intermediation*

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**Abstract**

Why do markets participants obtain funds through a repurchase agreements (repos) than by selling the asset spot? What determine the repo haircuts? What is the role played by re-use of the collateral sold to the lender? To answer these questions, we characterize the properties of the repurchase agreements (repos) traded in equilibrium. We show that a repo allows investors to borrow against their asset holdings while insuring both borrowers and lenders against future market price risk. Repos on safer assets command a lower haircut and a higher liquidity premium relative to riskier assets. If collateral is–scarce, haircuts may also be negative. We show that traders benefit from re-using the collateral sold in a repo. First, re-use allows the economy to sustain more borrowing with the same quantity of asset, thus generating a "collateral multiplier" effect. Second, with

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collateral re-use, lenders might also choose to re-pledge the asset to third parties. We characterize the conditions under which intermediation arises as an equilibrium choice of traders. These findings are helpful to rationalize chains of trades observed on the repo market.

# 1 Introduction

Gorton and Metrick (2012) make the case that the financial panic of 2007-08 started with a run on the market for repurchase agreements (repos). Their paper was very influential in shaping our understanding of the crisis. It was quickly followed by many attempts to understand repo markets more deeply, both empirically and theoretically, as well as calls to regulate these markets.[1]

A repo is the sale of an asset combined with a forward contract that requires the original seller to repurchase the asset at a given price. Repos are different from simple collateralized loans in (at least) one important way. A repo lender obtains the legal title to the pledged collateral and so gains the option to use the collateral during the length of the forward contract. This practice is known as re-use or re-hypothecation.[2] This special feature of repos has attracted a lot of attention from economists and regulators alike.

Repos are extensively used by market makers and dealer banks as well as

[1]See Acharya (2010) "A Case for Reforming the Repo Market" and (FRBNY 2010)

[2]Aghion and Bolton (1992) argue that securities are characterized by cash-flow rights but also control rights. Collateralized loans grant neither cash-flow rights nor control rights over the collateral to the lender unless the counterparties sign an agreement for this purpose. As a sale of the asset, a repo automatically gives the lender full control rights over the security as well as over its cash-flows. Re-use rights follow directly from ownership rights. As Comotto (2014) explains, there is a subtle difference between US and EU law however. Under EU law, a repo is a transfer of the security's title to the lender. However, a repo in the US falls under New York law which is the predominant jurisdiction in the US. *"Under the law of New York, the transfer of title to collateral is not legally robust. In the event of a repo seller becoming insolvent, there is a material risk that the rights of the buyer to liquidate collateral could be successfully challenged in court. Consequently, the transfer of collateral in the US takes the form of the seller giving the buyer (1) a pledge, in which the collateral is transferred into the control of the buyer or his investor, and (2) the right to re-use the collateral at any time during the term of the repo, in other words, a right of re-hypothecation. The right of re-use of the pledged collateral (...) gives US repo the same legal effect as a transfer of title of collateral."* To conclude, although there are legal differences between re-use and rehypothecation, they are economically equivalent (see e.g. Singh, 2011) and we treat them as such in our analysis.

other financial institutions as a source of funding, to acquire securities that are on specials, or simply to obtain a safe return on idle cash. They are closely linked to market liquidity and so they are important to understand from the viewpoint of Finance. Major central banks around the globe use repos to steer the short term nominal interest rate. The U.S. Federal Reserve recently introduced reverse repos to better control short term rates. Repos thus became essential to the conduct of monetary policy. Finally, firms use collateralized borrowing and some forms of repos to finance their activities or hedge exposures (notably interest rate risk, see BIS, 1999). This affects real activities, and so repos are also an important funding instrument for the macroeconomy.

Most existing research papers study specific aspects of the repo markets, e.g. exemption from automatic stay, fire sales, etc., taking the repo contract and most of its idiosyncrasies as given. These theories leave many fundamental questions unanswered, such as why are repos different from collateralized loans? What is the nature of the economic problem repo contracts are trying to solve? To answer these questions, to understand the repo market and the effect of regulations, one cannot presume the existence or the design of repo contracts.

In this paper we present a simple model where we derive the properties of the equilibrium repos. Traders prefer these contracts to spot trades and to collateralized contracts that do not allow for re-use.

The model has three periods and two types of investors, a natural borrower and a natural lender, both risk-averse, who lack the technology to commit to future promises. The borrower is endowed with an asset that yields an uncertain payoff in the last period. The payoff realization becomes known in the second period and is then reflected in the price of the asset. To increase his consumption in the first period, the borrower could sell the asset to the lender in the spot market. However this trade will expose both parties to price risk in the second period. Instead, the borrower can obtain resources from the lender by selling the asset combined with a forward contract promising to repurchase the asset in period 2. Unlike in an outright sale, a constant repurchase price in a repo hedges market price risk. Under limited commitment however, the borrower may find it optimal to default if the value of collateral falls below the promised repayment. We assume the punishment for default is the loss of the asset pledged as collateral together with a penalty that may depend on the borrower's characteristics and reflects his creditworthiness. To avoid default, the promised repayment should not be too high relative to the market value of the asset.

Both a borrowing and a hedging motives determine the repo contract traded in equilibrium. When the market value of the asset is low, the borrower cannot promise to repay much, as he would otherwise default, thus limiting his borrowing capacity. In contrast, when the market value of the asset is high, the repurchase price of the equilibrium contract is constant thus hedging both investors.

Using this equilibrium contract we derive comparative statics for haircuts and liquidity premia. Haircuts increase with counterparty risk, as a riskier investor can promise less income per unit of asset pledged. More risky collateral commands a higher haircut and a lower liquidity premium. Compared to a safe asset, a risky security pays less in bad times and more in good times. Since investors are constrained in bad times, this is precisely when collateral is valuable. Hence the liquidity premium is higher for the safe asset. In good times however, investors do not exploit the higher value of the riskier collateral since the repurchase price becomes constant. Hence, compared to the safe asset, less of the risky asset payoff is pledged and the haircut is larger.

In Section 4, we introduce collateral re-use. In a repo, the lender indeed acquires ownership of the asset used as collateral in the transaction. In our model, investors always choose to re-use the asset pledged as collateral whenever they have this option. To fix ideas suppose the collateral is perfectly safe, sells for \$100 in the first period, and also pays \$100 in the second period. Suppose the borrower has one unit of collateral and can promise to repay say up to \$110 per unit of the asset. So he obtains \$110 in a first round of repo with the lender. The lender can then re-use some of the collateral by selling it back to the borrower. The latter uses some of his \$110 to purchase that amount of collateral and can now pledge another \$110 per unit. With one round of re-use, the borrower netted an extra \$10 per unit. These trades can be repeated until no collateral may be re-used. In this simple example, the haircut is negative, but as we show in the text, a similar result obtains when haircuts are positive. Overall, re-use has a multiplier effect since a borrower can pledge more income per unit of asset in those states where he is constrained. We show that this collateral multiplier effect depends on the recourse nature of repo transactions. There is no such multiplier when the only punishment for default is the loss of collateral. Overall, the model implies that collateral re-use should be more prevalent for assets that command low haircuts and when the lender's trading partners have low counterparty risk.

Finally, Section 5 discusses the implications of collateral re-use for the repo market structure. We argue that some participants naturally emerge as intermediaries when they can re-use collateral. In practice, dealer banks indeed make for a significant share of this market by intermediating between natural borrowers (say hedge funds) and lenders (say money market funds or MMF). This might seem puzzling if direct trading platforms are available for both parties to bypass the dealer bank.[3] Our model rationalizes intermediation with difference in counterparty quality and ability to re-deploy the collateral. In our example, the hedge fund delegates borrowing to the dealer bank if the later is more creditworthy. Although there are larger gains from trade with the MMF, the hedge fund prefers borrowing from the dealer bank if he is more efficient at re-using collateral. Indeed, through re-use, one unit pledged to the dealer bank can then support more borrowing in the chain of transactions. Our model thus provides an endogenous theory for repo intermediation based on fundamental heterogeneity between traders.

**Relation to the literature**

Gorton and Metrick (2012) argue that the recent crisis started with a run on repo whereby funding dropped dramatically for many financial institutions. Subsequent studies by Krishnamurty et al. (2014) and Copeland et al. (2014) have qualified this finding by showing that the run was specific to the - large - bilateral segment of the repo market. Recent theoretical works indeed highlighted some features of repo contracts as sources of funding fragility. As a short-term debt instrument to finance long-term assets, Zhang (2014) and Martin et al. (2014) show that repos are subject to roll-over risk. Antinolfi et al. (2015) emphasize the costs and benefits of the exemption from automatic stay for repo collateral. Lenders easy access to the borrower's collateral may be privately optimal but collectively harmful in the presence of fire sales, a point also made by Infante (2013) and Kuong (2015).

[3]In the US, Direct Repo$^{TM}$ provides this service

These papers usually take repurchase agreements as given while we want to understand their emergence as a funding instrument. One natural question is to ask why borrowers do not simply sell the collateral to lenders? A first strand of papers explains the existence of repos using search frictions (e.g. Narajabad and Monnet, 2012, Tomura, 2013, and Parlatore, 2015). Bundling the sale and the repurchase of the asset in one transaction lowers search costs or mitigates bargaining inefficiencies. Bigio (2015) and Madison (2016) emphasize asymmetry of information about the quality of the asset to explain repos. There, the debt-like feature of a repo contract reduces adverse selection between the informed seller and the uninformed buyer as in DeMarzo and Duffie (1999) or Hendel and Lizzeri (2002). We show that repos exists in an environment with symmetric information, where investors trade contracts on a Walrasian market, but where the collateral has uncertain payoff. Our theory also rationalizes haircuts since borrowers choose repos when they could obtain more cash in the spot market.[4] In addition, we account for the sale of collateral in a repo by considering re-use.

To derive the repo contract, we follow the competitive approach of Geanakoplos (1996) , Araújo et al. (2000) and Geanakoplos and Zame (2014) where collateralized promises traded by investors are selected in equilibrium. Unlike these papers where the only cost from default is the loss of the collateral, our model aims to capture the recourse nature of repo transactions. We thus allow for a partial recovery of the shortfall and an extra penalty for default in the spirit of Dubey et al. (2005). While our results on the design of repo contracts carry through without this penalty, the recourse nature of repos is crucial to explain re-use. Indeed, Maurin (2015) showed in a more general environment that the collateral multiplier effect disappears if loans are non-recourse.

In the second part of the paper, we account for the transfer of the legal title to the collateral to the lender, opening the possibility for re-use. Singh and Aitken (2010) and Singh (2011) argue that collateral re-use lubricates transactions in the financial system.[5] However re-use introduces the risk that the collateral taker does not or cannot return the collateral as explained by Monnet (2011). Unlike

[4]In particular, we do not need transactions costs as suggested by Duffie (1996).

[5]Fuhrer et al. (2015) estimate an average 5% re-use rate in the Swiss repo market over 2006-2013.

Bottazzi et al. (2012) or Andolfatto et al. (2014), we thus account for the limited commitment problem of the collateral taker, when studying the benefits of re-use. Asset re-use resembles pyramiding (see Gottardi and Kubler, 2015) whereby a newly issued debt claim is used as collateral. Collateral re-use differs because of the two sided limited commitment problem which is absent with pyramiding. In addition, while pyramiding merely allows for a more efficient use of collateral, re-use has a multiplier effect. We stress the role of collateral re-use in explaining repo market intermediation as in Infante (2015) and Muley (2015). Unlike these papers, intermediation arises endogenously in our model as trustworthy investors re-use the collateral from risky counterparties to borrow on their behalf. In an empirical paper, Issa and Jarnecic (2016) indeed suggested that the fee based view of repo intermediation whereby dealers gain from differences in haircuts does not stand in the data.

The structure of the paper is as follows. We present the model and the complete market benchmark in Section 2. We analyze the optimal repo contracts, including properties for haircuts, liquidity premiums, and repo rates in Section 3. In Section 4, we allow for collateral re-use and study intermediation in Section 5. Finally, Section 6 concludes.

# 2 The Model

In this section we present a simple environment where risk averse investors have fundings needs to smooth their income. These investors can trade securities in a competitive financial market, but limited commitment requires that borrowing is backed by some collateral. investors may trade the asset spot or simple collateralized contracts without re-use, but in equilibrium, they will choose to trade repos.

## 2.1 Setting

The economy lasts three periods, $t = 1, 2, 3$. There are two types of investors $i = 1, 2$ and one consumption good each period. Both investors have endowment $\omega$ in all but the last period. Investor 1 is also endowed with $a$ units of an asset while investor 2 has none.[6] This asset pays dividend $s$ in period 3. The dividend is distributed according to a cumulative distribution function $G(.)$ with support $\mathcal{S} = [\underline{s}, \bar{s}]$ and with mean $E[s] = 1$. The realization of $s$ becomes known to all investors in period 2. As a consequence, price risk arises in period 2.

Let $c_t^i$ denote investor $i$'s consumption in period $t$. Preferences from consumption profile $(c_1^i, c_2^i, c_3^i)$ for investor $i = 1, 2$ are:

$$\begin{aligned} U^1(c_1^1, c_2^1, c_3^1) &= c_1^1 + v(c_2^1) + c_3^1 \\ U^2(c_1^2, c_2^2, c_3^2) &= c_1^2 + u(c_2^2) + \beta c_3^2 \end{aligned}$$

where $\beta < 1$, $u(.)$ and $v(.)$ are respectively strictly concave and concave functions. We assume $u'(\omega) > v'(\omega)$ and $u'(2\omega) < v'(0)$, so that there are gains from transferring resources from investor 1 to investor 2 in date 2 and the optimal allocation is interior. These preferences contain two important elements. First, as $\beta < 1$, investor 2 values less consumption in date 3, so investor 1 is the natural holder of the asset in that period. Second, investors with strictly concave utility function dislike consumption variability in period 2.

## 2.2 Arrow-Debreu equilibrium

Here we show that the Arrow-Debreu equilibrium allocation $(\mathbf{c}_*^1, \mathbf{c}_*^2)$ is not contingent on the realization of the asset's dividend, a feature we use extensively later on. In this economy, $(\mathbf{c}_*^1, \mathbf{c}_*^2)$ is characterized by equal marginal rates of substitution unless one investor is at a corner. We guess and verify that this is the case between the first and the second period and we obtain the following equilibrium conditions:

$$\begin{cases} u'(c_{2,*}^2) = v'(2\omega - c_{2,*}^2) \\ c_{3,*}^2 = 0 \end{cases} \tag{1}$$

where we used the resource constraint of period 2 to substitute for $c_{2,*}^1 = 2\omega - c_{2,*}^2$. Intuitively, since $\beta < 1$, investor 2 does not consume in period 3 because he has a lower marginal utility than investor 1. The implicit prices for period 2 and 3

[6] This is for simplicity only and we could easily relax this assumption, as none of the results depend on it.

consumption are respectively $u'(c^2_{2,*})$ and 1. To pin down the equilibrium allocation completely, we use the budget constraint of investor 2 and derive his period 1 consumption $c^2_{1,*} = \omega - u'(c^2_{2,*})(c^2_{2,*} - \omega)$. This expression is positive if :

$$\omega \geq u'(c^2_{2,*})(c^2_{2,*} - \omega) \tag{2}$$

which we assume in the remainder of the text. In equilibrium, investor 1 borrows $c^2_{2,*} - \omega$ at a net interest rate $r^* = 1/u'(c^2_{2,*}) - 1$. In the following we refer for simplicity to the Arrow-Debreu equilibrium allocation as the first best allocation. Observe that consumption in period 2 $(c^1_{2,*}, c^2_{2,*})$ is deterministic although the asset payoff $s$ is already known. Indeed, risk averse investors prefer a smooth consumption profile.

## 2.3 Financial Markets With Limited Commitment

While investors want to engage in borrowing and lending, they may not be able to fully commit to future promised payments. Hence the first best equilibrium allocation cannot be sustained and borrowing positions must be collateralized. investors can trade their asset spot or they can trade financial securities in zero net supply in a competitive market.

*Spot Trades*

The spot market price in period 1 is denoted $p_1$ and the price in period 2 and state $s$ is $p_2(s)$ which reflects the future known payoff $s$ of the asset. Let us denote $a^i_1$ (resp. $a^i_2(s)$) the asset holdings of investor $i$ after trading in period 1 (resp. period 2 and state $s$). Using spot trades, investor 2 can implicitly lend to investor 1 if he buys the asset in period 1, that is $a^2_1 > 0$ and re-sells it in period 2, that is $a^2_2(s) < a^2_1$. In the Appendix we show that a combination of spot trades alone can never sustain the first best allocation because $p_2(s)$ is a function of the state which generates undesirable consumption variability in period 2 for both investors.

*Repos*

In addition to spot transactions, investors can also trade in period 1 securities that are essentially debts or promises to deliver the consumption good in period 2. We let $f = \{f(s)\}_{s \in \mathcal{S}}$ denote the payoff schedule for a generic security. An investor selling $f$ promises to repay $f(s)$ in state $s$ of period 2 per unit of security.

We allow for all possible values of $f(s)$ so that the market for financial securities is complete. Short positions are backed by collateral as otherwise investors may default. Without loss of generality, an investor must post one unit of collateral per unit of security sold. The asset is a financial claim – and not a real asset –, which makes it possible for the lender to re-use the collateral pledged. However, collateral re-use introduces a double commitment problem.

In what follows we specify what happens to the collateral and the punishment for default to capture the main features of repo contracts. The ownership of the collateral is transferred to the lender who is able to re-use the collateral. Specifically, investor $i$ can re-use a fraction $\nu_i$ of the collateral he receives where $\nu_i \in [0, 1]$. We interpret $\nu_i$ as a measure of the operational efficiency of a trader in re-deploying collateral for his own trades.[7] The other fraction $1 - \nu_i$ is segregated.

When facing a default, a creditor can seize the asset used as collateral, which he can hold or sell in the spot market. In addition, he recovers a fraction $\alpha \in [0, 1]$ of the shortfall, that is the difference between the promised repayment and the market value of the collateral. Finally, a defaulting investor $i$ incurs a non-pecuniary cost equal to a fraction $\pi_i \in [0, 1]$ of the contractual repo payment, measured in consumption units.

As specified, the securities match several features of repo contracts. First, they are loans collateralized by financial asset. Second, the lender gets possession of the collateral since it is sold by the borrower. He may then sell it when the borrower defaults but also re-use the asset pledged during the lifetime of the transaction.[8] Finally, repos are recourse-loans. Under the most popular master agreement described in ICMA (2013), an investor can indeed claim the shortfall to a defaulting counterparty in a "close-out" process. Our partial recovery rate $\alpha$ captures the monetary value of delay or other impediments in recouping this shortfall. The non-pecuniary component proxies for legal and reputation costs or losses from future market exclusion.[9] The parameter $\pi$ may depend on the identity of the borrower.

[7] Singh (2011) discusses the role played by collateral desks at large dealer banks in channeling these assets across different business lines. These desks might not be available for less sophisticated repo market participants such as money market mutual funds or pension funds. In practice, the bulk of traded repos have short maturity, limiting the scope for re-use.

[8] While a repo is not characterized as a sale in the US, the exemption from automatic stay for repo collateral gives similar rights for the lender. See also footnote 2 on this point.

[9] The functional form will ensure that prices are linear function of trades. We thus depart

We allow the repo repurchase price $f(s)$ to be state contingent, a natural feature in our environment. This might be viewed as unrealistic since repos usually specify a fixed repayment. But note that margin calls or repricing of the terms of trade during the lifetime of a repo are ways in which contingencies can arise.[10] In Section 6, we also discuss contracts with fixed repayment to show that our main results hold qualitatively.

*Borrower and Lender Default*

In a repo, the borrower promises to repay the lender who pledges to return the collateral. Hence, a dual limited commitment problem arises. To explicit each counterparty incentives to default, consider a trade of one unit of repo contract $f$ between borrower $i$ and lender $j$. This comes without loss of generality because penalties for default are linear in the amount traded.

Borrower $i$ prefers to repay rather than default if and only if:

$$f(s) \leq p_2(s) + \alpha(f(s) - p_2(s)) + \pi_i f(s) \tag{3}$$

The left hand side is the repurchase price of the asset. For the borrower to repay, $f(s)$ must not exceed the total default cost. The first term is the loss of the market value $p_2(s)$ of the collateral seized by the lender. The second term $\alpha(f(s) - p_2(s))$ is the fraction of the shortfall recovered by the lender. The third term $\pi_i f(s)$ is the non-pecuniary cost for the borrower. Notice that default is only meaningful when $\pi_i + \alpha < 1$ and we concentrate on this case from now on.

We now turn to the lender's incentives to return the asset.[11] Recall that he can only re-use a fraction $\nu_j$ of the collateral. We assume that he deposits or segregates the non re-usable fraction $1 - \nu_j$ with a collateral custodian. As a result, he may only abscond with the re-usable fraction of the collateral.[12] When

from most models of collateralized lending a la Geanakoplos (1996) which assume $\alpha = \pi = 0$. As we argued, our assumptions seem natural for repos which are recourse loans.

[10] When he faces a margin call, a trader must pledge more collateral to sustain the same level of borrowing. This is equivalent to reducing the amount borrowed per unit of asset pledged.

[11] Technically, most Master Agreements characterize as a "fail" and not an outright default the event where the lender does not return the collateral immediately. While our model does not distinguish between fails and defaults, lenders also incur penalties when they fail.

[12] It is easy to understand why this is optimal for him ex-ante. First, he is less likely to default ex-post. Second, by definition, he would not derive ownership benefits from keeping the non re-usable collateral on his balance sheet. In the tri-party repo market, BNY Mellon and

the lender defaults, the borrower gets the $1-\nu_j$ units of segregated collateral back. He also recovers a fraction $\alpha$ of the shortfall $p_2(s)-f(s)-(1-\nu_j)p_2(s)$, symmetrically with the case of a borrower's default. Hence, the lender prefers to return the re-usable collateral rather than default if and only if

$$\nu_j p_2(s) \leq f(s) + \alpha(\nu_j p_2(s) - f(s)) + \pi_j f(s) \tag{4}$$

The left hand side is the benefit of defaulting and keeping the re-usable units of collateral evaluated at market value.[13] The right hand side is the cost of defaulting. The first term is the foregone payment $f(s)$ from the borrower. The lender also loses the fraction $\alpha$ of the shortfall $\nu_j p(s)-f(s)$ which is recovered by the borrower. Finally, he incurs the non-pecuniary cost $\pi_j f(s)$.

Our model has several implications for the cost and benefit of default. First, the non-pecuniary punishment generates a deadweight loss. This should encourage investors to trade default-free contracts. However, investors may want to trade default-prone contracts because borrowers can indirectly pledge the endowment $\omega$ through the recovery payment (if $\alpha > 0$) when they default. We show in the Proof of Proposition 1 that focusing on default-free contracts comes without loss of generality when the following condition holds:

$$\pi v'(\omega) \geq \alpha(u'(\omega) - v'(\omega)) \tag{5}$$

Intuitively, repo contracts inducing defaults are dominated if the marginal cost of default $\pi v'(\omega)$ exceeds the marginal benefits $\alpha(u'(\omega)-v'(\omega))$ through the pecuniary transfer with the recovery of the shortfall.

We can now define the set of no-default repo contracts $\mathcal{F}_{ij}$ between two investors $i$ and $j$ as a function of the period 2 spot market price $\mathbf{p}_2 = \{p_2(s)\}_{s\in\mathcal{S}}$. To simplify notation, we let $\theta_i := \pi_i/(1-\alpha)$. Transforming equations (3) and (4), we obtain

JP Morgan provide these services. Our results extend with some modification to the case where segregation is not available. Essentially, the no-default constraint of the lender might become binding for high values of $s$, while it is not in our baseline specification.

[13] A lender might re-use collateral and not have in on his balance sheet when he must return it to the lender. However, observe that he can purchase the relevant quantity of the asset in the spot market to satisfy his obligation. When he returns the asset, the lender effectively covers a short position $-\nu_j$.

the set of no-default repos.

$$\mathcal{F}_{ij}(\mathbf{p}_2) = \left\{ f \mid \forall\, s \in [\underline{s}, \bar{s}]\,,\ \frac{\nu_j p_2(s)}{1+\theta_j} \leq f(s) \leq \frac{p_2(s)}{1-\theta_i} \right\} \tag{6}$$

Since investor $i$ is less likely to default when $\theta_i$ is high, we interpret this parameter as a measure of creditworthiness or counterparty quality. Observe that the set $\mathcal{F}_{ij}(\mathbf{p}_2)$ is convex and that prices are linear functions of quantity traded. In addition, we normalized all contracts by unit of asset pledged. Hence, for any combination of multiple contracts sold by $i$, there exists an equivalent trade of a single repo contract. In the following, we thus call without ambiguity $f_{12}$ and $f_{21}$ the equilibrium contracts.

We denote $q_{ij}(f_{ij})$ the price of the contract $f_{ij} \in \mathcal{F}_{ij}$ traded by investors $i$ and $j$. When indexing a contract, the subscript $ij$ reflects the equilibrium choice of repos by investors $i$ and $j$.[14] For simplicity, we write $q_{ij} := q(f_{ij})$ and refer to $q_{ij}$ as the repo price.

*investors optimization problem.*

We can now write the investors' optimization problem. Given prices, investors are choosing which contract to trade and the volume of trade for that contract. We formalize the equilibrium choice of the repo contract below in Definition (2.3). We let $b^{ij}$ (resp. $l^{ij}$) denote the amount investor $i$ borrows (resp. lends) with $j$ using equilibrium contract $f_{ij}$ (resp. $f_{ji}$).

$$\max_{a_t^i, b^{ij}, l^{ij}} E\left[U^i(c_1^i, c_2^i(s), c_3^i(s))\right] \tag{7}$$

subject to

$$c_1^i = \omega + p_1(a_0^i - a_1^i) + q_{ij}b^{ij} - q_{ji}l^{ij} \tag{8}$$

$$c_2^i(s) = \omega + p_2(s)(a_1^i - a_2^i(s)) - f_{ij}(s)b^{ij} + f_{ji}(s)l^{ij} \tag{9}$$

$$c_3^i(s) = a_2^i(s)s \tag{10}$$

$$a_1^i + \nu_j l^{ij} \geq b^{ij} \tag{11}$$

$$b^{ij} \geq 0 \tag{12}$$

$$l^{ij} \geq 0 \tag{13}$$

[14]The subscript $ij$ also indexes the price to the extent that investors may have different re-use abilities.

At date 1, investor $i$ has resources $\omega + p_1 a_0^i$ and chooses asset holding $a_1^i$, lending $\ell^{ij}$ and borrowing $b^{ij}$. Given these decisions, his resources at date 2 is the endowment $\omega$ and the value of his asset holdings $p_2(s)a_1^i$ as well the net value of the repo positions $f_{ji}(s)\ell^{ij} - f_{ij}(s)b^{ij}$. Equation (11) is the collateral constraint of investor $i$. When investor $i$ borrows, that is $b^{ij} > 0$, he must hold one asset per unit of repo contract sold. He can buy these assets either in the spot market, that is $a_1^i > 0$ or in the repo market if $l^{ij} > 0$. In the latter case, however, only a fraction $\nu_j$ of the asset purchased can be re-used.

For later reference, it is important to note from the collateral constraint that a lender can take a short position on the spot market. Let indeed $b^{ij} = 0$ and $l^{ij} > 0$. Then, it can be that $a_1^i < 0$ if $\nu_i > 0$. With re-use, a lender acquires ownership of the asset pledged by the lender and can then sell it to create a short-position. Indeed, when the repo matures, investor 2 would then have the obligation to return an asset that he does not hold anymore. The only difference with a regular sale is that the lender who acquired the asset in a repo is committed to return the asset to the borrower.

**Definition.** ***Repo equilibrium***

*An equilibrium is a system of spot prices $p_1$ and $\mathbf{p_2} = \{p_2(s)\}_{s\in\mathcal{S}}$, a pair of repo contracts $(f_{12}, f_{21}) \in \mathcal{F}_{12}(\mathbf{p_2}) \times \mathcal{F}_{21}(\mathbf{p_2})$, their prices $q_{12}$ and $q_{21}$, and allocations $\{c_t^i(s), a_t^i, \ell^{ij}, b^{ij}\}_{t=1..3,s\in\mathcal{S}}^{i=1,2.j\neq i}$ such that*

1. *$\{c_t^i(s), a_t^i, \ell^{ij}, b^{ij}\}_{t=1..3,s\in\mathcal{S}}^{j\neq i}$ solves investor $i = 1, 2$ problem (7)-(13).*
2. *Markets clear, that is $a_2^1 + a_1^2 = a$ and $b^{ij} = l^{ji}$ for $i = 1, 2$ and $j \neq i$*
3. *For any contract $\tilde{f} \notin \{f_{12}, f_{21}\}$, there exists a price $q_{ij}(\tilde{f})$ such that investors do not trade this contract.*

Points 1 and 2 are self-explanatory. Point 3 formalizes the optimality condition for the choice of contracts. A repo contract can be part of an equilibrium if and only if investors do not wish to trade an alternative contract $\tilde{f}$. For example, if $\tilde{f} \in \mathcal{F}_{12}(\mathbf{p_2})$, the implicit equilibrium price $q(\tilde{f})$ must be too low (resp. too high) for investor 1 (resp. investor 2) to wish to sell (resp. to buy) this contract. Hence, with our equilibrium definition, all contracts are available to trade and investors select their preferred contracts taking prices as given.

# 3 Repo markets with no re-use

In this section, we characterize the equilibrium when investors cannot re-use collateral, that is $\nu_1 = \nu_2 = 0$. Then, a repo contract is a standard collateralized loan taken by investor 1. Since only the contract $f_{12}$ will be traded in equilibrium, we simplify notation by setting $f = f_{12}$.

## 3.1 Equilibrium repo contract

To gain intuition, remember that, at the first best allocation, investor 1 borrows in period 1 by promising to repay $c^2_{2,*} - \omega$ in period 2. In a repo equilibrium and given $p_2(s)$, the maximum pledgeable income of investor 1 is $\frac{ap_2(s)}{1-\theta_1}$. This expression obtains when investor 1 sells all his asset in a repo, that is $b^{12} = a$, with the highest possible repurchase price $p_2(s)/(1-\theta_1)$. In low states, this income may fall short of $c^2_{2,*} - \omega$ and the repurchase price should indeed be set as high as possible because gains from trade are not exhausted. In high states however, this could raise investor 2 consumption too much. There, the repurchase price $f(s)$ should be constant. We let $s^*$ be the threshold between these two regions. Formally, it is the solution to

$$c^2_{2,*} = \omega + \frac{ap_2(s^*)}{(1-\theta_1)} = \omega + \frac{as^*}{v'(c^1_{2,*})(1-\theta_1)}. \tag{14}$$

The second equality follows from the observation that $p_2(s) = s/v'(c^1_2(s))$ in equilibrium, since investor 1 holds the asset into period 3. At $s^*$ investor 1 can just finance the first-best allocation if he pledges his entire wealth. Observe that $s^*$ is decreasing with $a$ and $\theta_1$. So it is easier to achieve the first best allocation the larger the stock of asset and the more creditworthy investor 1 is. Given the repo trade above, we can now determine the equilibrium $p_2(s)$ as the unique solution – increasing in $s$ – to

$$\begin{cases} p_2(s)v'\left(\omega - a\dfrac{p_2(s)}{1-\theta_1}\right) = s & \text{if} \quad s < s^* \\ p_2(s)v'(c^1_{2,*}) = s & \text{if} \quad s \geq s^* \end{cases} \tag{15}$$

We have the following result.

**Proposition 1.** *There is a unique equilibrium allocation where investors trade repo contract $f$ characterized as follows:*

1. *If $s^* \geq \bar{s}$ (a is low), $f(s) = p_2(s)/(1-\theta_1)$ for all $s \in \mathcal{S}$*

2. *If $s^* \in [\underline{s}, \bar{s}]$ (a is intermediate),*

$$f(s) = \begin{cases} \dfrac{p_2(s)}{1-\theta_1} & \text{for } s \leq s^* \\ \dfrac{p_2(s^*)}{(1-\theta_1)} & \text{for } s \geq s^* \end{cases} \tag{16}$$

3. *If $s^* \leq \underline{s}$ (a is high), $f(s) = f^*$ for all $s \in \mathcal{S}$ where $f^* \in [\frac{p_2(s^*)}{(1-\theta_1)}, \frac{p_2(\bar{s})}{(1-\theta_1)}]$.*

*where $p_2$ is defined in (15). In equilibrium, investors strictly prefer to trade repo over any combination of repo and spot trades in cases 1 and 2. They are indifferent between both in case 3.*

The equilibrium contract reflects both investor 1 desire to borrow in period 1 and the aversion to the payoff risk in period 2. As we explained, investor 1 can pledge at most $p_2(s)/(1-\theta_1)$ per unit of asset in state $s$. This amount increases in $s$ together with the collateral value $p_2(s)$. When the collateral value is low, for $s \leq s^*$, the borrowing constraint of investor 1 is binding and the repurchase price $f(s)$ is equal to the maximum pledgeable income for investor 1. This borrowing motive explains why $f(s)$ is increasing in $s$ for $s \leq s^*$. However, when the collateral value is high, investor 1 does not want to increase the income pledged over the first best amount. Hence, the repurchase price becomes flat for $s \geq s^*$. As a result, consumption is constant thus hedging investors against the price risk for states $s \geq s^*$. We call this the hedging motive. Given this repo contract, we show in the Appendix that investors do not want to trade any other contracts. Figure 1 plots the equilibrium repo contract, in the case $v(x) = \delta x$ for $\delta \in (0, 1)$.

It is interesting to emphasize why investors prefer trading repo rather than spot. Suppose indeed that investor 1 sells the asset spot in period 1 and buys it back at the spot market price $p_2(s)$ in period 2. This is formally equivalent to a

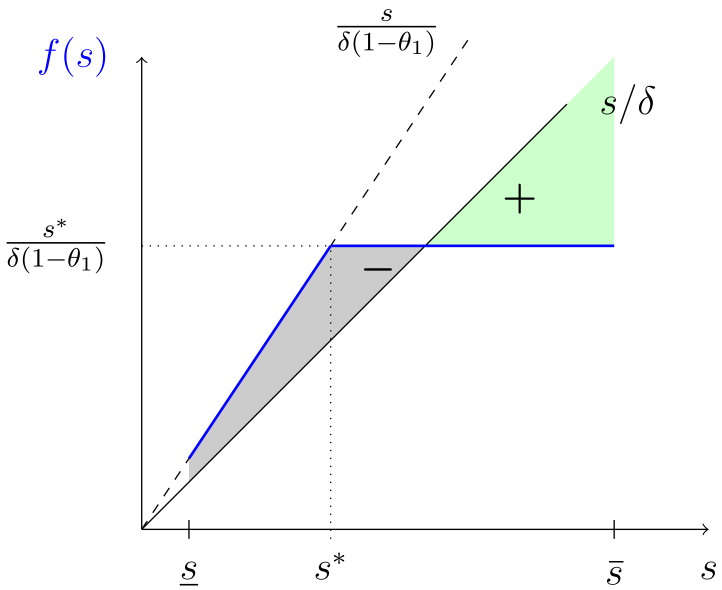


Figure 1: Repo contract ($v(x) = \delta x$).

repo contract $\hat{f}$ with $\hat{f}(s) = p_2(s)$. This alternative trade is dominated for two reasons. When the collateral value is low, investor 1 can increase the amount he pledges from $p_2(s)$ to $p_2(s)/(1-\theta_1)$ with a repo. When the collateral value is high, the hedging motive insures investors against the price risk of spot trades with a flat repurchase price.

We can associate the equilibrium repurchase price to the repo rate $r$ where:

$$1 + r = \frac{\mathbb{E}[f(s)]}{q} = \frac{\mathbb{E}[f(s)]}{\mathbb{E}[f(s)u'(c_2^2(s))]}. \tag{17}$$

When investors are constrained (case i) and ii) of Proposition 1), we have $u'(c_2^2(s)) > u'(c_{2,*}^2)$ for $s \in [\underline{s}, s^*]$ so that $1 + r < 1 + r^*$. investor 2 would like to lend at the frictionless interest rate $1 + r^*$. However, investor 1 cannot increase borrowing since he runs out of collateral. The interest rate must then fall for investor 2 to be indifferent.

## 3.2 Haircuts and liquidity premium

In this section, we derive the equilibrium properties of the liquidity premium and repo haircut. We define the liquidity premium $\mathcal{L}$ as the difference between the spot price of the asset in period 1 and its fundamental value.[15] We thus obtain

$$\mathcal{L} \equiv p_1 - E[s]$$

The liquidity premium is also the shadow price of the collateral constraint. It thus captures the value of the asset as an instrument to borrow over and above its holding value. Hence, whenever the asset is scarce and investors are constrained, the asset bears a positive liquidity premium. Using the equilibrium characterization, we can relate the liquidity premium to the payoff of the repo contract and the marginal utilities of the borrowers and lenders:

$$\mathcal{L} = E[f(s)(u'(c_2^2(s)) - v'(c_2^1(s))]$$

When $s^* \leq \underline{s}$, investors are not constrained and $c_2^2(s) = c_{2,*}^2$ for all $s$, and $\mathcal{L} = 0$. When $s^* > \underline{s}$, we have $u'(c_2^2(s)) > v'(c_2^1(s))$ for $s < s^*$ , that is some gains from trade are not realized in low states because repo collateral is scarce. The strictly positive liquidity premium reflects the scarcity: it is equal to the average of the repurchase price multiplied by the wedge in marginal utilities.

The repo haircut is the difference between the spot market price and the repo price. Indeed, it costs $p_1$ to obtain 1 unit of the asset, which can be pledged as collateral to borrow $q$. So to purchase 1 unit of the asset, an investor needs $p_1 - q$ which is the downpayment or haircut.[16]

$$\mathcal{H} \equiv p_1 - q = E[(p_2(s) - f(s))v'(c_2^1(s))] \tag{18}$$

where the second equality follows from the first order condition of investor 1 with respect to spot and repo trades. As Figure 1 shows, the borrowing and hedging motives have opposite effects on the size of the haircut. In the states $s < s^*$ where

[15] Formally, the fundamental value of the asset is its price in the Arrow-Debreu equilibrium.

[16] An alternative but equivalent definition is $(p_1 - q)/q$.

investors are constrained, the borrower uses the maximum pledgeable capacity $p_2(s)/(1-\theta_1)$ per unit while the asset trades at price $p_2(s)$. From expression (18), this contributes negatively to the haircut. However, in states $s \geq s^*$ , investor 1 does not use the full collateral value of the asset. In particular, the repayment $f(s)$ is flat while the asset value $p_2(s)$ increases with $s$. This contributes positively to the haircut. The overall sign of the haircut depends on the weights on both regions in the distribution of $s$. Finally, observe that the haircut is not pinned down when $s^* \leq \underline{s}$ since several (constant) repurchase prices $f$ are possible in equilibrium.

#### 3.2.1 Collateral scarcity and counterparty quality

In this section we derive comparative statics for the liquidity premium and haircut relative to the scarcity of collateral and the counterparty quality.

**Proposition 2.** *$\mathcal{L}$ is decreasing and $\mathcal{H}$ is increasing in the amount of collateral $a$. $\mathcal{L} = 0$ whenever $a$ is large enough that investors can reach the FB allocation in all states, that is $s^* \leq \underline{s}$. $\mathcal{H}$ decreases in counterparty quality $\theta_1$ while the effect on $\mathcal{L}$ is ambiguous.*

When $a$ increases, there is more asset to use as collateral in a repo. investor 1 can thus borrow more in states $s < s^*$, which reduces the wedge $u'(c_2^2(s)) - v'(c_2^1(s))$ in marginal utilities. The liquidity premium, which is the shadow price of collateral, geos down as more gains from trade are realized. Haircuts increase with $a$ because $s^*$ goes down as the quantity of asset $a$ increases. Hence, there are less states where the repurchase price contributes negatively to the haircut.

A higher counterparty quality $\theta_1$ decreases haircuts since the pledgeable capacity $p_2(s)/(1-\theta_1)$ increases. Intuitively, a better counterparty has a higher ability to honor debt, which reduces the downpayment. Figure 2 illustrates the effect of an increase from $\theta_{1L}$ to $\theta_{1H} > \theta_{1L}$. The solid line representing the borrowing capacity shifts to the left. This naturally leads to a decrease in the haircut, by increasing the size of the region where $f(s) > p_2(s)$ while leaving the other region unchanged.

When it comes to the liquidity premium $\mathcal{L}$, counterparty quality $\theta_1$ has an ambiguous effect. First, an increase in $\theta_1$ allows investor 1 to borrow more[17] in states

[17] Although the effect is intuitive, the effect of an increase of $\theta_1$ on the pledgeable amount

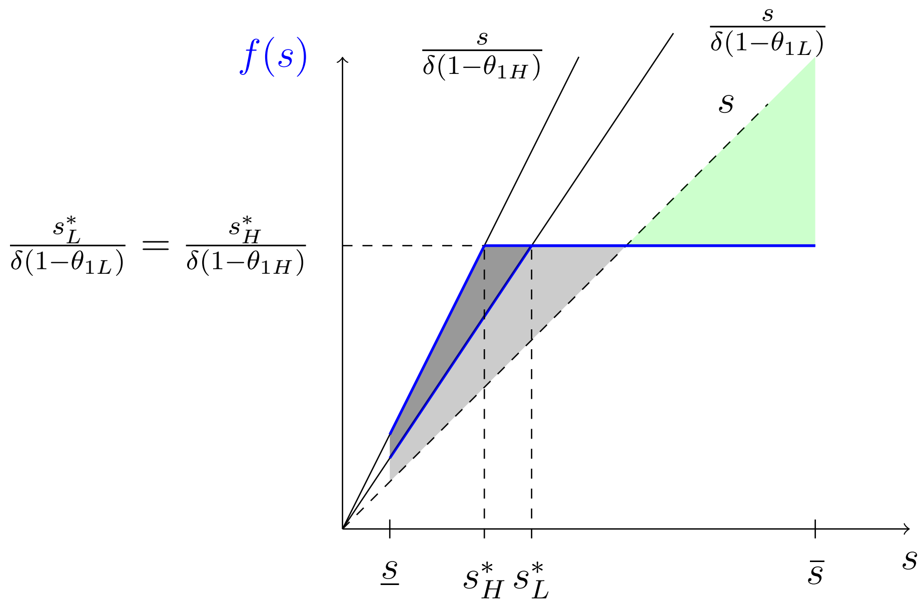


Figure 2: Influence of $\theta$, with $\theta_H > \theta_L$ $(v(x) = \delta x)$

$s < s^*$, which reduces the wedge $u'(c_2^2(s)) - v'(c_2^1(s))$ in marginal utilities. This effect, similar to an increase in the asset available $a$, tends to reduce the liquidity premium. However, conditional on a value of $s^*$, $\theta_1$ determines the repurchase price schedule while $a$ does not. Indeed, on those states where the investors are constrained, the repo contract $f(s)$ is equal to the maximum pledgeable capacity $p_2(s)/(1-\theta_1)$. As more income can be pledged when this is most valuable, the asset becomes a better borrowing instrument, which raises its price. Thus, counterparty quality $\theta_1$ can have a non-monotonic impact on the liquidity premium $\mathcal{L}$.

### 3.2.2 Asset risk

Our model allows us to analyze the relationship between haircuts and liquidity premium of two assets with different risk profile. We characterize the equilibrium

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$p_2(s)/(1-\theta_1)$ is not straightforward. Remember indeed that the spot market price $p_2(s)$ is pinned down by the relationship $p_2(s)v'(\omega - ap_2(s)/(1-\theta_1)) - s = 0$ for $s \leq s^*$ so that $p_2(s)$ decreases with $\theta_1$. However, one can easily show that the net effect on the pledgeable income is positive, that is $\partial[p_2(s)/(1-\theta_1)]\partial\theta_1 > 0$.

when investors can trade both assets at the same time rather than comparing quantities across equilibria with a single asset.[18] The extent to which investors are constrained, that is the marginal utility wedge $u'(c_2^2(s)) - v'(c_2^1(s))$ for any $s$, is then the same for both assets. As our exercise effectively controls for market conditions, we think it is more meaningful to bring it to the data. To make things simple, we introduce two assets with perfectly correlated payoffs but different risk to ignore the effect of risk sharing on the structure of the repo contract.

As before, $s \sim G[\underline{s}, \bar{s}]$ but there are now two assets $i = A, B$ with payoffs $\rho_i(s)$:

$$\rho_i(s) = s + \alpha_i(s - \mathbb{E}[s]),$$

where $\alpha_B > \alpha_A = 0$. With $\alpha_A = 0$, asset $A$ is our benchmark asset. Since $\alpha_B > 0$, asset $B$ has the same mean but a higher variance than asset $A$. Indeed $Var[\rho_\alpha] = (1 + \alpha)Var[s]$. investor 1 is endowed with $a$ units of asset $A$ and $b$ units of asset $B$, while investor 2 does not hold any of the assets. It is relatively straightforward to extend the equilibrium analysis of the previous section to this new economy. The set of available contracts consists of feasible repos using assets $A$ and $B$. For each asset $i = A, B$, the repo contract $f_i$ uses the maximum pledgeable capacity up to the state where the first best level of consumption can be reached. We then prove the following result.

**Proposition 3.** *The safer asset $A$ always has a higher liquidity premium and a lower haircut than the riskier asset $B$.*

The key intuition behind the result is the misallocation of collateral value across states induced by a mean preserving spread. Asset $A$ and $B$ have the same expected payoff. However, since $\rho_B(s) - \rho_A(s) = \alpha_B(s - E[s])$, the risky asset pays relatively more in high states (upside risk) and less in low states (downside risk). The collateral is valuable when investors are constrained, that is in low states. Since the safe asset $A$ pays more in these states, it carries a larger liquidity premium. We now turn to the haircut. In high states, the riskier asset $B$ has a higher payoff which means that more income can be pledged compared to asset $A$.

[18] For the sake of completeness, we also computed the comparative statics related to a mean preserving spread. It implies a higher haircut, but the effect on the liquidity premium is indeterminate and depends on risk aversion.

However, investor 1 does not wish to borrow over the first best level in those states. Since less of the risky asset's payoff is pledged, the haircut is larger. Observe that without the hedging motive, asset risk would have no impact on the haircut.

So far, repos are indistinguishable from standard collateralized loans. Indeed, with $\nu = 0$, the asset is immobile once pledged in a repo. The next two sections show that allowing for re-use delivers new predictions. First, re-use increases the borrowing capacity of investor 1. Second, the possibility to re-use collateral may lead to endogenous intermediation in equilibrium.

# 4 The multiplier effect of re-use

In this section, we analyze the impact of collateral re-use on equilibrium contracts and allocations. This is a natural feature of a repo trade where the collateral is sold to the lender. Many have discussed the consequences of re-use in repo contracts (see Singh and Aitken, 2010). Our model allows to precisely characterize the benefits of re-use and the effects on repo contracts. The lender, investor 2 is now able to re-use collateral, that is $\nu_2 > 0$ while for simplicity we maintain $\nu_1 = 0$ and we discuss the consequences of $\nu_1 > 0$ in a remark, later in the text.

To understand the potential benefits, consider the equilibrium without re-use. In the first rounds, investor 1 pledges all his asset as collateral in a repo with the lender. At this stage, investor 2 (the lender) holds $a$ units of collateral. Allowing for re-use frees up a fraction $\nu_2$ of this collateral. Let us consider the following pattern of trades. After this first round, investor 2 sells $\epsilon$ units to investor 1 (where $\epsilon$ is small). Investor 1 purchases this asset and pledges it in a second round of repo with the same terms. By definition of equilibrium, the marginal gain for investor 1 from this trade is null since it is already feasible without re-use. The marginal gain to investor 2 is:

$$
\begin{aligned}
\frac{\partial U^2}{\partial \epsilon} &= p_1 - E[p_2(s)u'(c_2^2(s))] - q + E[f(s)u'(c_2^2(s))] \\
&= \frac{\theta_1}{1-\theta_1} \int_{\underline{s}}^{s^*} p_2(s) \left( u'(c_2^2(s)) - v'(c_2^1(s)) \right) dF(s)
\end{aligned}
$$

where we derive the second equality in the Appendix. Hence, this marginal gain is strictly positive when $s^* > \underline{s}$ (investors are constrained) and $\theta_1 > 0$. To understand this last condition, notice that investor 2 gets $\frac{p_2(s)}{1-\theta_1}\epsilon$ in state $s$ of period 2 from the second round of repo, but has to purchase $\epsilon$ units of asset to return the full collateral on the first repo. When investor 2 resells some collateral he receives from investor 1, he effectively short-sells the asset. So the net additional transfer to investor 2 in period 2 for all $s < s^*$ where investors are constrained is

$$-p_2(s)\epsilon + \frac{p_2(s)}{1-\theta_1}\epsilon = \frac{\theta_1}{1-\theta_1}p_2(s)\epsilon$$

This transfer is positive and increases investor 1's borrowing only if $\theta_1 > 0$. In all other states $s > s^*$ gains from trade are exhausted, so the marginal impact of the net transfer is null. These steps can be repeated over multiple rounds. At the end of the second round, investor 2 has $\epsilon$ units of the asset from which he can re-use $\nu_2\epsilon$ and sell it to investor 1. investor 1 would then pledge an additional $\frac{\theta_1}{1-\theta_1}\nu_2\epsilon p_2(s)$ in state $s$. After this operation, investor 2 has $(\nu_2)^2\epsilon$ units of re-usable asset. Iterating over these rounds infinitely, the total pledgeable amount per unit of asset in state $s$ obtains:

$$\begin{aligned} M_{12}p_2(s) :&= \frac{p_2(s)}{1-\theta_1} + \sum_{i=1}^{\infty}(\nu_2)^i \frac{\theta_1}{1-\theta_1}p_2(s) \\ &= \frac{1}{1-\nu_2}\left[\frac{1}{1-\theta_1} - \nu_2\right]p_2(s) \end{aligned} \tag{19}$$

where we call $M_{12}$ the borrowing multiplier, that is the pledgeable amount normalized for the value of one unit of the asset. The borrowing multiplier is strictly increasing in $\nu_2$ as long as $\theta_1 > 0$. The multiplier $M_{12}$ and the asset quantity held by investor 1 determine his borrowing capacity with investor 2.

Re-use however induces two changes to the original contract. First, it lowers $s^*$: the borrowing multiplier increases the number of states where investors can attain the first best since more income can be pledged for a given quantity of collateral. Using the multiplier, we can define $s^*(\nu_2)$ as the minimal state above which investor 1 can pledge enough income to finance the first best allocation, that is:

$$\omega + aM_{12}p_2(s^*(\nu_2)) = c^2_{2,*}.$$

The second change in the structure of the repo contract comes from the short position that investor 2 builds when he re-uses the collateral (see the discussion preceding the definition of a repo equilibrium). To unwind his short position, investor 2 has to purchase the asset in the spot market in period 2, which exposes him to price risk. The repo contract will seek to correct this additional risk. As before, when $s < s^*(\nu_2)$ the borrowing motive dominates the hedging motive, so that the structure of the contract does not change. But for $s > s^*(\nu_2)$ the contract will reflect the cost for investor 2 of unwinding his short position $\nu_2 p_2(s)$ as keeping $f(s)$ constant in that range would make investor 2 suffer the price risk.

We can then introduce the candidate equilibrium repo contract $f(s;\nu_2)$ where:

$$f(s,\nu_2) = \begin{cases} \dfrac{p_2(s)}{1-\theta_1} & \text{if } s < s^*(\nu_2) \\ \dfrac{s^*(\nu_2)}{(1-\theta_1)v'(c^1_{2,*})} + \dfrac{\nu_2(s-s^*(\nu_2))}{v'(c^1_{2,*})} & \text{if } s \geq s^*(\nu_2) \end{cases} \tag{20}$$

In general, when re-using collateral ($\nu_2 > 0$), the lender could default on his promise to return the asset. However, contract (20) satisfies the no-default constraint of the lender (4) for any value of $\nu_2$; the payment from the repo contract $f(s,\nu_2)$ is always higher than the value of the re-usable collateral $\nu_2 p_2(s)$. The following Proposition establishes that investors trade this contract in an equilibrium with re-use:

**Proposition 4.** ***Collateral Re-use.*** *Let $\nu_1 = 0$, $\nu_2 \in (0,1)$, $\theta_1 > 0$, and $s^*(0) = s^* > \underline{s}$ (the first-best allocation cannot be achieved without re-use). There is a unique equilibrium allocation where investor 1 borrows using repo contract $f(\nu_2)$ defined in (20) and investors 2 re-sells collateral in equilibrium. There exists $\nu^* < 1$ such that for $\nu_2 > \nu^*$ investors reach the first-best allocation.*

As we discussed before, when $\theta_1 > 0$, re-use strictly increases the amount investor 1 can pledge to investor 2. This is valuable when investors are constrained and want to expand borrowing in low states. From the expression of $M_{12}$ in (19), it is clear that for $\nu_2$ high enough, the first-best allocation can even be financed in the lowest state $\underline{s}$. One can obtain the expression for $\nu^*$ by setting $s^*(\nu_2) = \underline{s}$. Some simple algebra in the Appendix shows

$$\nu^* = \frac{s^* - \underline{s}}{s^* - (1-\theta_1)\underline{s}}.$$

Proposition (4) shows that investors always want to re-use collateral when they can, and this result holds independently of the sign of the haircut. If the haircut is negative, it is intuitive that re-use is beneficial to the natural borrower, investor 1. Indeed, by buying 1 unit of asset from investor 2, investor 1 can pledge it back in a repo which yields him a net gain of $-p_1 + p_F = -\mathcal{H}$ in period 1. This increases the consumption of investor 1 whenever $\mathcal{H} < 0$. When $\mathcal{H} > 0$ it may seem that investor 1 loses from re-use. But this logic is incomplete since investor 1 may also gain by transferring consumption across states in period 2. If the haircut is positive, an incremental amount of collateral re-use decreases investor 1 consumption in period 1, but it smoothes his consumption across states in period 2, as he consumes more in the high states and less in the low states. We show that the second effect always dominates the first when $\mathcal{H} > 0$.

The liquidity premium $\mathcal{L}$ can exhibit non-monotonicity in the re-use factor $\nu_2$. While re-use relaxes the collateral constraint, it also increases the amount pledgeable in states where investors are constrained. This last effect makes the asset more valuable and can increase the liquidity premium. These two effects are reminiscent of the comparative statics with respect to counterparty quality $\theta$. Finally, our model predicts that the benefits of re-use are larger when collateral is most scarce (that is $s^* > \underline{s}$) and there is evidence that this is indeed the case (see Fuhrer et al., 2015).

*Remark. Re-use through repo vs. spot sales ($\nu_1 > 0$)*

Since investor 2 is the natural lender, it seems that the re-use capacity of the borrower $\nu_1$ should play no role. But recall that investor 2 is free to re-use a fraction of the collateral in a spot sale or in a repo where he would borrow from investor 1. investor 1 is willing to engage in a repo as a lender as long as he can re-use a high enough fraction of the asset to increase his borrowing. Proposition 4 assumes that $\nu_1 = 0$ so that any collateral pledged to investor 1 becomes immobile. As a consequence, investor 2 was only re-using the asset in a spot sale rather than in a repo. We now argue that when $\nu_1$ is sufficiently large, investor 2 prefers to re-use the asset in a repo instead. This is the case when the marginal increase in pledgeable income by investor 1 is larger when investor 2 chooses this option, or

$$-\frac{\nu_1}{1+\theta_1}p_2(s) + \nu_1 M_{12} p_2(s) \geq -p_2(s) + M_{12} p_2(s).$$

The right hand side measures the net increase in pledgeable income when investor 2 sells spot, that we derived earlier. The left hand side is the equivalent expression for a repo sale by investor 2 where $f_{21}(s) = \nu_1 p_2(s)/(1+\theta_1)$. This is the minimal value of $f_{21}$ that will induce investor 1 not to default as a repo buyer. Buying in a repo is thus less costly than buying spot for investor 1 since $\nu_1/(1+\theta_1) < 1$. This increases the net transfer to investor 2 per unit of the asset. However, investor 1 can only re-use $\nu_1$ units of the asset to sell in a repo. The same multiplier is applied to this fraction $\nu_1$ to compute the second term. Elementary transformations of this inequality yields the following condition.

$$\frac{2\nu_1(1-\nu_2)}{(1-\nu_2\nu_1)(1+\theta_1)} > 1 \tag{21}$$

Intuitively, when $\nu_1$ is sufficiently large, re-using through repos generates a larger increase in pledgeable income. The equilibrium characterization in Proposition 4 we stated for $\nu_1 = 0$ thus remains valid when inequality (21) is not satisfied. When (21) holds,[19] investor 2 re-sells in a repo to investor 1.

# 5 Collateral Re-use and Intermediation

In their guide to the repo market, Baklanova et al. (2015) state that "*dealers operate as intermediaries between those who lend cash collateralized by securities, and those who seek funding*". To fix ideas, let us consider the following chain of

[19] In equilibrium, this will also affect the repo contract sold by investor 1 to investor 2. Although the equilibrium contracts change, the core intuition remains. Collateral re-use allows investor 2 to sell the asset back to investor 1, whether spot or repo, for him to increase the amount he borrows.

trades. First, a hedge fund who needs cash borrows from a dealer bank through a repo. The dealer bank then taps in a money market fund (MMF) cash pool through another repo to finance the transaction. Figure 3 illustrates this pattern of repo intermediation. Since direct platforms such as Direct Repo™ in the US are available, why do traders engage in repo intermediation? In this section, we explain these chain of repos based on different counterparty quality for hedge fund and the dealer bank.[20] A remarkable feature of our analysis is that intermediation arises endogenously although in our example, the hedge fund would be free to trade directly with the MMF.[21]

We extend the economy slightly for this purpose, introducing a third type of investor named $B$, for dealer Bank. investor $B$ has no asset initially. He is endowed with $\omega$ in period 1 and 2 and has the following preferences:

$$U^B(c_1, c_2, c_3) = c_1 + \delta_B c_2 + c_3$$

For simplicity, we assume here that investor 1 also has linear preferences, that is $v(x) = \delta x$ or :

$$U^1(c_1, c_{2,} c_3) = c_1 + \delta c_2 + c_3$$

We let $\delta \leq \delta_B < u'(\omega)$. In this environment, investor $B$ also wants to borrow from investor 2 but he has lower gains from trade than investor 1. We set $\theta_B > \theta_1$ so that the Bank has a higher creditworthiness than investor 1. The corresponding greater borrowing capacity per unit of asset will explain why investor $B$ can play a role as an intermediary. All investors are free to participate in the spot market and engage in repo trades with any type of counterparty. We will say that there is intermediation when investor 1 sells his asset only to $B$ and that $B$ re-sells to investor 2. For simplicity, we set $\nu_1 = 0$ in this Section.

[20] In practice, the transaction between the dealer bank and the MMF could take place using a Tri-Party investor as a custodian. We abstract from modeling the services provided by the Tri-Party investor. See Federal Reserve Bank of New York (2010) for a discussion of Tri-Party repo. We thus focus on the intermediation provided by the dealer bank to the hedge fund and the MMF.

[21] This result extends Infante (2015) and Muley (2015) which assume repo trades must occur through intermediaries.

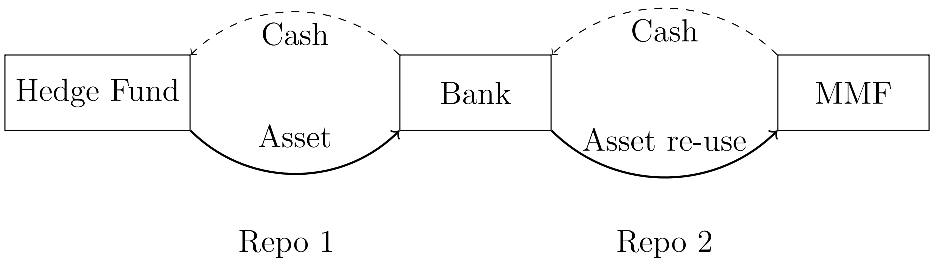


Figure 3: Intermediation with Repo

## 5.1 Intermediation via spot trades

We assume first that investors 1 and B have the same preferences, that is $\delta = \delta_B$ and only differ in their creditworthiness. Although investors 1 and B have no gains from trade, we show that in equilibrium, the latter plays a role as an intermediary.

**Proposition 5.** *Let $\delta = \delta_B$ and $\theta_1 < \theta_B$. There exists $a^*$ such that when $a < a^*$, the equilibrium is as follows. investor 1 sells his asset spot. investor $B$ buys the asset spot which he sells in a repo to investor 2.*

The striking feature in Proposition 5 is that investor 1 who is endowed with the asset does not trade a repo with investor 2, the natural lender. Instead, in equilibrium, investor 1 sells the asset spot to $B$. Once investor $B$ acquires the asset, he finds himself in the same position as investor 1 in the last Section vis a vis investor 2. In particular, with re-use, the equilibrium repo contract $f_{B2}$ is similar[22] to (20), replacing $\theta_1$ with $\theta_B$.

Without investor $B$, investor 1 would borrow in a repo from investor 2 as before. However, investor $B$ may pledge more income to investor 2 due to its higher creditworthiness. In a competitive equilibrium, investor $B$ makes no profit as an intermediary. Hence, the benefits from the higher borrowing capacity with investor 2 is fully reflected in the spot price he pays for the asset to investor 1. As

[22] As before, investors $B$ and 2 do multiple rounds of repo since investor 2 may re-use the asset. investor 2 re-sells repo rather than spot the collateral pledged by investor $B$ if :

$$\frac{2\nu_B(1-\nu_2)}{(1-\nu_2\nu_B)(1+\theta_B)} > 1$$

This expression is the same as (21) replacing $\nu_1$ by $\nu_B$ and $\theta_1$ by $\theta_B$.

a result, investor 1 now prefers to sell his asset in the spot market, thus delegating borrowing to a more creditworthy investor.

When $\delta = \delta_B$, intermediation takes place via a spot trade between investors 1 and $B$ and not via a repo. Observe indeed that there are no direct gains from trade between 1 and $B$. As a result, they do not value the extra borrowing capacity from a repo when $\theta_1 > 0$. To the contrary, trading repo is costly because a fraction $1 - \nu_B$ of the asset could not be used by investor $B$ to borrow from 2. A trade-off emerges when $\delta < \delta_B$ and, as we show in the next subsection, a chain of repos may therefore emerge in equilibrium.

The upper bound $a^*$ on the quantity of asset $a$ is the threshold below which the first best allocation with $u'(c^2_{2,*}) = \delta$ is not attainable. When $a \geq a^*$, other trade patterns are possible in equilibrium. In particular, investor 1 holds enough asset to attain the first best allocation with investor 2 despite his low creditworthiness. In this case, investor $B$ could be inactive. An interesting implication of our result is thus that intermediation should be observed precisely when collateral is scarce.

## 5.2 Chain of repos

We now let $\delta < \delta_B$ and show that an equilibrium with a chain of repos may exist. When $\delta_B - \delta$ is sufficiently large, the larger gains from trade generated by the extra borrowing capacity of the repo sale may compensate the costs attached to the collateral segregation.

If investors trade in a chain of repo, investor $B$ acts both as a lender with investor 1 and as a borrower vis a vis investor 2. This creates a competing use for the asset. When he holds one unit of it, investor $B$ may either sell the asset back to investor 1 for him to increase borrowing or use it to borrow from investor 2. We will show that in equilibrium, $B$ should be indifferent between the two usages.

Finally, investor 1 must prefer trading in a repo with investor $B$ with whom direct gains from trade are smaller than with investor 2. This pattern may arise if indirect gains from trade with $B$ through re-use are larger. Indeed, one unit of pledged collateral can be redeployed at different rates by counterparties with different re-use ability $\nu$. In particular, we have shown that the multiplier between borrower $i$ and lender $j$ is:

$$M_{ij} = \frac{1}{1-\nu_j}\left[\frac{1}{1-\theta_i} - \nu_j\right] \quad j = B, 2 \tag{22}$$

In equilibrium, investor 1 will prefer to trade with $B$ if the larger borrowing multiplier compensates for the lower direct gains from trade.

We call intermediation equilibrium with a chain of repos an equilibrium where the following pattern of trades is observed: investor 1 sells the asset in a repo to investor $B$ who re-uses the asset to sell in a repo to investor 2. We may now state the exact conditions under which a chain of repo arises in equilibrium.

**Proposition 6. *Intermediation equilibrium.***

*Let $\nu_2 = 0$. An intermediation equilibrium with a chain of repos exists iff $\delta_B - \delta$ is neither too small, nor too large and*

$$1 - \nu_B \leq \frac{\theta_B - \theta_1}{1 - \theta_1} \tag{23}$$

*investors 1 sells the asset repo to $B$ where $f_{1B}$ is given by*

$$f_{1B}(s) = \frac{s}{1-\theta_1} \qquad \forall s \in [\underline{s}, \bar{s}] \tag{24}$$

*investor $B$ sells the asset repo to 2 where for some $s^*_{B2} \in [\underline{s}, \bar{s}]$, $f_{B2}$ is given by*

$$f_{B2}(s) = \begin{cases} \frac{p_2(s)}{1-\theta_B} & \text{if } s < s^*_{B2} \\ \frac{p_2(s^*)}{1-\theta_B} & \text{if } s \geq s^*_{B2} \end{cases}$$

Observe first that the repo contract $f_{1B}$ between investors 1 and $B$ does not reflect any hedging motive since both investors are risk neutral. For investors $B$ and 2, the repo contract is essentially the same as in the previous section with $\nu_2 = 0$.

We characterize the lower and upper bounds on $\delta_B - \delta$ in the Appendix. To form intuition about these conditions, we assume that $s = 1$ in our discussion. When involved in a chain of repos, investor $B$ acquires re-usable collateral from investor 1. He may either re-resell it to investor 1 or re-pledge it to investor 2. In equilibrium, he must be marginally indifferent between these two options. Suppose for instance that he strictly prefers to re-pledge the collateral to investor 2. Then, some asset that has been segregated in the repo trade between 1 and $B$ is misallocated and should rather support trade between $B$ and 2. As a result, investor 1 would sell some of his asset spot to investor $B$. This indifference condition can be written as follows:

$$M_{B2}[u'(c_2^2) - \delta_B] = (M_{1B} - 1)[\delta_B - \delta] \tag{25}$$

for some $c_2^2 - \omega \in [0, \nu_B a]$, the consumption level of investor 2 in period 2. The left hand side is the gain from selling the asset in a repo to investor 2, that is the marginal benefit times the borrowing multiplier $M_{B2}$ between the two investors. The right hand side is the gain from re-selling the asset spot to investor 1. The term between parenthesis $M_{1B} - 1$ is the borrowing multiplier between 1 and $B$ net of the cost of acquiring the asset for investor 1. The lower and upper bounds on $\delta_B - \delta$ obtain from evaluating the indifference condition (25) at $c_2^2 - \omega = \nu_B a$ and $c_2^2 - \omega = 0$ respectively. When investor $B$ employs all the collateral available to re-pledge to investor 2, we have indeed $c_2^2 - \omega = \nu_B a$. When he re-uses only by re-selling to investor 1, we have $c_2^2 - \omega = 0$.

We now discuss condition (23). Observe that in equilibrium, investor 1 sells the asset in a repo with investor $B$ rather than with investor 2. We argue that the first option indeed dominates the second if:

$$\frac{\delta_B - \delta}{1 - \theta_1} + \nu_B M_{B2}(u'(c_2^2) - \delta_B) \geq \frac{u'(c_2^2) - \delta}{1 - \theta_1}$$

where $c_2^2 - \omega \in [0, \nu_B a]$. The left hand side (resp. right hand side) measures the gains from selling the asset in a repo to investor $B$ (resp. 2). The first component on each side of the inequality captures the gains from trade ignoring the possibility of re-use. These are larger with investor 2 since $u'(c_2^2) > \delta_B$. However, with re-use, there are also indirect gains from trading with $B$ since he can redeploy the collateral (the second term on the left hand side). These gains are not present with investor 2 when he cannot re-use collateral ($\nu_2 = 0$).[23] The possibility to re-use

[23] In the appendix, we show that the result also holds when $\nu_2$ is positive but sufficiently smaller than $\nu_B$.

collateral thus explains why seemingly dominated trades (here between 1 and $B$) can take place.[24] Equilibrium condition (23) obtains by plugging equality (25) into the inequality above. We can read (23) as a cost benefit analysis of intermediation. The cost on the left hand side is the fraction of collateral segregated. The benefit on the right hand side is the (normalized) extra borrowing capacity $\theta_B - \theta_1$ of investor $B$ with respect to 1.

To summarize, an investor may become a dealer if he is more creditworthy than the natural borrower and more efficient at re-deploying collateral than the natural lender. Our analysis thus shows that repo intermediation arises endogenously out of fundamental heterogeneity between traders. Existing models of repo intermediation typically take the chain of possible trades as exogenous. Our approach is helpful to rationalize several features of the repo market. First, we can explain why intermediating repo is still popular despite the emergence of direct trading platforms. Second, in exogenous intermediation models, dealers typically gain and collect fees by charging higher haircuts to borrowers. In our model, the haircut paid by the borrower to the bank may very well be smaller than the one paid by the bank to the lender. Using data from the Australian repo market, Issa and Jarnecic (2016) show that this is indeed the case in most transactions.

# 6 Fixed Repurchase Price

In this section, investors can only trade contracts with a fixed repurchase price. This can be viewed as more realistic for short-term maturity repos where margin call or repricing do not occur (see the discussion in Section 2.3). We claim that under some additional condition, investors still prefer trading repo than spot and value the ability to re-use the collateral. To simplify the analysis, we consider the

[24] Condition (23) is equivalent to

$$M_{1B}(\delta_B - \delta) \geq M_{12}(u'(c_2^2) - \delta)$$

From the point of view of investor 1, borrowing from investor $B$ dominates if the multiplier $M_{1B}$ is larger than $M_{12}$ although gains from trade are smaller ($\delta_B - \delta \leq u'(c_2^2) - \delta$). Again this is possible only if $\nu_B > \nu_2$.

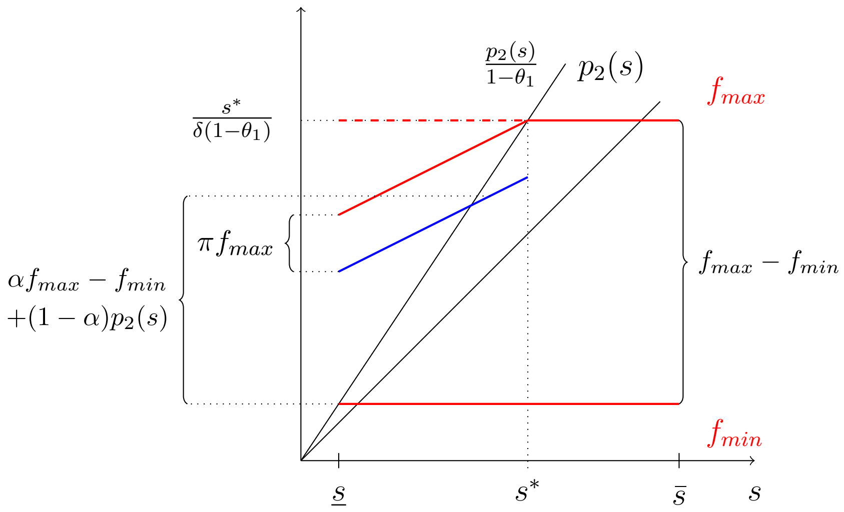


Figure 4: Repo contracts with fixed repayment

two investors case with linear preference for investor 1 that is:

$$U^1(c_1, c_{2,}c_3) = c_1 + \delta c_2 + c_3$$

Let now $f$ denote both a repo contract and its fixed promised payoff across states. Since a risky asset with value $p_2(s)$ now backs a promise of a fixed payment $f$, a trade-off arises between the borrowing capacity and the cost of default. To gain intuition, remember that with state-contingent repurchase price, default never arises in equilibrium under Assumption 5. In any given state $s$, the deadweight cost exceeded the benefit from pledging income through the partial recovery of the shortfall. When repurchase prices are fixed however, a marginal increase in $f$ has an additional effect. First, as before, the seller is now more likely to default in low states which is costly. However, it also increases the amount pledged in high states. This is valuable if gains from trade are not exhausted. When the second effect dominates, equilibrium default can be profitable.

We illustrate this trade-off on Figure 4 using two extreme contracts. Repo $f_{min}$ is the the maximum repurchase price such that investor 1 never defaults. Repo $f_{max} \geq f_{min}$ is the repurchase price that finances the first best allocation in states

$s \geq s^*$. We thus have

$$f_{min} = \frac{\underline{s}}{\delta(1-\theta_1)}, \qquad f_{max} = \frac{s^*}{\delta(1-\theta_1)}$$

Contract $f_{max}$ induces default in states $s \leq s^*$. The cost to the borrower, represented by the solid red line, is equal to $\alpha f_{max} + (1-\alpha)p_2(s) + \pi f_{max}$. The benefit to the borrower is represented by the solid blue line. It is obtained from a downward parallel shift of $\pi f_{max}$ - the deadweight cost of default - from the red line. Observe first that investors do no want to trade a contract $\tilde{f} \notin [f_{min}, f_{max}]$. A lower repurchase price than $f_{min}$ just reduces the borrowing capacity. In turn, a repurchase price above $f_{max}$ will induce default in some states $s > s^*$ without any benefit since gains form trade would be exhausted above $s^*$. To understand the costs and benefits of defaults with fixed repurchase price, suppose that investor 1 sells all the asset in repo $f_{min}$ to investor 2. We thus have $c_2^2 = \omega + af_{min}$. Consider a marginal deviation whereby investor 1 re-allocates $\epsilon$ collateral units from $f_{min}$ to support borrowing using $f_{max}$. On states $s \leq s^*$, investor 1 defaults on $f_{max}$ so that the net marginal benefit in state $s \leq s^*$ is

$$[\alpha f_{max} + (1\text{-}\alpha)p_2(s) - f_{min}]\left(u'(c_2^2) - \delta\right) - \pi\delta f_{max}$$

The first term is the net gain from pledging more income through default. The second term is the deadweight cost. The sign of this expression is ambiguous. Under Assumption 5, the expression above is negative in neighborhood of $\underline{s}$ but it can be positive close to $s^*$. On states $s \geq s^*$, investor 1 marginally increases the amount pledged without default. The positive benefits in any state $s \geq s^*$ are equal to:

$$(f_{max} - f_{min})(u'(c_2^2) - \delta)$$

Intuitively, a repo traded in equilibrium should balance the cost of default with the strength of the borrowing motives.

The general characterization of equilibrium with fixed repurchase prices is difficult for two reasons. First, the trade-off described above depends on the distribution $G$ of the asset payoff. Second, with default and fixed repurchase price, several repos may be traded in equilibrium. Indeed, the equilibrium payoff set is not convex anymore.[25] Still, we claim that if the following condition is satisfied:

$$\frac{\underline{s}}{1-\theta_1} > E[s] \tag{26}$$

then, there exists a contract $f \in (f_{min}, f_{max})$ with equilibrium default such that investor 1 strictly prefers to sell repo rather than spot. As a corollary, investors 1 and 2 benefit from a marginal re-use of the collateral. We prove this claim in the Appendix. To simplify our analysis, we derive the equilibrium trades with spot trades and the given repo $f$ but not the equilibrium repo(s). Essentially, we drop the third requirement from Definition 2.3 which means that deviations to other repo trades could be profitable[26].

It is not straightforward anymore that investors strictly prefer repo over spot. A combination of spot trades cannot be replicated with a fixed repurchase repo. Condition (26) ensures that investor 1 strictly prefers to sell the asset in the no-default repo $f_{min}$ than spot. Intuitively, it must be that $f_{min} = \underline{s}/(1-\theta_1)$ allows to borrow more on average than the value of the asset $E[s]$. The result that investors would also prefer a repo contract with default $f > f_{min}$ over a spot trade is by continuity at $\underline{s}$.

The second point is a direct consequence of the first. We have seen previously that when investor 1 strictly prefers to trade repo than spot, investor 2 short sale constraint binds. Hence, investor 2 is willing to re-sell collateral for investor 1 to borrow more in the repo, at least marginally.

This result emphasizes again the role of recourse in repo transactions to explain the benefits from collateral re-use. If loans are non-recourse, that is $\theta_1 = 0$, condition (26) cannot hold. Hence, our results do not rely on the possibility to adjust the terms of a repo (state-contingent repurchase price) but rather on the fact that repos are recourse loans.

[25] To be precise, for some non-trivial convex combination of two repos $f$ and $f'$, there does not exist be a repo $f''$ such that $f''$ delivers the same payoff to the borrower and the lender than the combination of $f$ and $f'$.

[26] It is in the spirit of our analysis of fixed repurchase price repos to also assume that there is some rigidity in setting the terms of a repo. Krishnamurty et al. (2014) provide evidence that lenders either lend or pull out their funding in the Tri-Party repo market but do not adjust terms of trade.

# 7 Conclusion

We analyzed a simple model of repurchase agreement with limited commitment and price risk. Unlike a combination of sale and repurchase in the spot market, a repo contract provides insurance against the asset price risk. We introduce counterparty risk as heterogenous cost from defaulting on the promised repurchase price. We showed that the repo haircut is an increasing function of counterparty risk and a decreasing function of the asset inherent risk. Safe assets naturally command a higher liquidity premium than risky ones. Our model targets repos rather than collateralized loans since we allow investors to re-use collateral. We showed that re-use increases borrowing through a collateral multiplier effect. In addition, it can explain intermediation whereby trustworthy investors borrow on behalf of riskier counterparties.

Our simple model delivers rich implications about the repo market but leaves many venues for future research. We argued that counterparty risk is a fundamental determinant for the terms of trade in repo contracts. It would be interesting to analyze the impact of clearing on repo market activity since clearing often implies novation by a central counterparty (see Mancini et.al. XXX). Novation bears some similarities with intermediation although terms of trades cannot be adjusted and risk may be concentrated on a single investor. When it comes to re-use, besides the limit on the amount of collateral that can be re-deployed, we assumed a frictionless process. Traders establish and settle positions smoothly although many rounds of re-use may be involved. This may not be the case anymore in the presence of frictions in the spot market for instance. Recent theoretical papers have shown that secured lending markets can be fragile. Although we did not investigate this aspect in the present work, we believe collateral re-use may add to this fragility.

# Appendices

## A Equilibrium analysis of spot trade only

We prove the following Proposition to characterize spot trade equilibria.

**Proposition 7.** When investors can only trade spot, there exists a threshold $\bar{a}_{spot}$ such that

1. Low asset quantity: if $a < \bar{a}_{spot}$, then investor 1 sells his entire asset holdings at date 1. The liquidity premium $\mathcal{L}$ is strictly positive.

2. High asset quantity:if $a \geq \bar{a}_{spot}$, then investor 1 sells less than $a$ at date 1. The liquidity premium is $\mathcal{L} = 0$.

Deriving the first order conditions, the following system of equations characterize the equilibrium.

$$
\begin{aligned}
c_3^1(s) &= \omega + as, \\
c_2^1(s) &= \omega - p_2(s)(a_1^2 - a_2^2(s)), \\
-p_2(s)v'(c_2^1(s)) &= s + \xi_2^1(s), && (27) \\
-p_2(s)u'(c_2^2(s)) &= \delta s + \xi_2^2(s), && (28) \\
-p_1 + E\left[p_2(s)v'(c_2^1(s))\right] + \xi_1^1 &= 0, \\
-p_1 + E\left[p_2(s)u'(c_2^2(s))\right] + \xi_1^2 &= 0, \\
\xi_1^1\xi_1^2 &= 0
\end{aligned}
$$

where $\xi_t^i$ is the Lagrange multiplier on the no-short sale constraint of investor $i$ in period $t$. Given that $u'(\omega) > v'(\omega)$, one can easily check that $\xi_2^1(s) = 0$ for all $s$. This is natural since investor 1 who does not discount period 3 payoffs is the natural holder of the asset. By the same logic, we have that $\xi_1^2 = 0$. investor 2 must buy a positive quantity of the asset since otherwise gains from trade are left on the table. From equation (27), it is easy to realize that $p_2(s)$ is increasing in $s$. Moreover there exists $\hat{s}(a_1^2)$ such that $a_2^2(s)$ is equal to 0 for $s \leq \hat{s}(a_1^2)$ and solves $p_2(s)u'(\omega + p_2(s)(a_1^2 - a_2^2(s)) = \delta s$ otherwise. investor 2 carries positive holdings of the asset into period 3 in those high states $s > \hat{s}(a_1^2)$ where re-selling everything would increase too much his period 2 consumption. Focusing now on period 1, we are left to pin down $a_1^2$, the quantity investor 2 initially buys from investor 1.

$$
\begin{aligned}
-p_1 + E\left[p_2(s)v'(c_2^1(s))\right] + \xi_1^1 &= 0, \\
-p_1 + E\left[p_2(s)u'(c_2^2(s))\right] &= 0, \\
c_2^1(s) &= \omega - p_2(s)a_1^2 \\
c_3^1(s) &= 2\omega + as
\end{aligned}
$$

so

$$\xi_1^1 = E\left\{p_2(s)\left[u'(c_2^2(s)) - v'(c_2^1(s))\right]\right\} \tag{29}$$

To solve for the equilibrium price $p_2(s)$ and the quantity sold $a_1^2$, let us introduce the following system:.

$$
\begin{aligned}
p_2(s)v'(\omega - p_2(s)a_1^2) &= s \\
K(a_1^2) &= E\left\{p_2(s)\left[u'(\omega + p_2(s)a_1^2) - v'(\omega - p_2(s)a_1^2)\right]\right\}
\end{aligned}
$$

The first equation implicitly defines $p_2(s)$ as a function of $s$ and $a_1^2$, using equation (27). The Implicit Function Theorem shows that $p_2(s)$ depends negatively on $a_1^2$. The total derivative of $K$ with respect to $a_1^2$ is equal to

$$K'(a_1^2) = \int_{\underline{s}}^{\hat{s}(a_1^2)} \left[\frac{\partial p_2(s)}{\partial a_1^2}\left\{u'(\omega + p_2(s)a_1^2) + a_1^2 p_2(s)u''(\omega + p_2(s)a_1^2)\right\} + p_2(s)^2 u''(\omega + p_2(s)a_1^2)\right] \mathrm{d}F(s)$$

This expression is strictly negative if the coefficient of relative risk aversion of $u$ is less than 1. Define then $\bar{a}_{spot}$ as the unique solution to $K(a_1^2) = 0$. Two cases are then possible: *i)* $a \geq \bar{a}_{spot}$ and $\xi_1^1 = 0$ and $a_1^2 = \bar{a}_{spot}$ or *ii)* $a < \bar{a}_{spot}$ and $\xi_1^1 > 0$

that is $a_1^2 = a$.

# B Proofs

## B.1 Proof of Proposition 1

In the absence of re-use ($\nu = 0$), the set of no-default repo contracts for investor $i \in \{1, 2\}$ at a given spot market price schedule $\boldsymbol{p_2} = \{p_2(s)\}_{s \in \mathcal{S}}$ is:

$$\mathcal{F}_i(\mathbf{p}_2) = \left\{ f \in \mathcal{C}^0[\underline{s}, \bar{s}] \mid 0 \leq f(s) \leq \frac{p_2(s)}{1 - \theta_i} \right\}$$

We proceed in three steps. First we characterize the equilibrium repurchase contract $f \in \mathcal{F}_1(\mathbf{p}_2)$ for a given spot price schedule $\mathbf{p}_2$, using the fact that investors must not be willing to trade any other feasible contract. Then we characterize the spot market price $\mathbf{p}_2$ compatible with the equilibrium. Finally, we back our claim that investors do not trade repo inducing default when assumption (5) holds. As we argued in the text, we need only consider a repo contract where investor 1 is the borrower that we call $f$ for simplicity. As we will show, there can be spot trades in equilibrium but they are redundant.

The equilibrium conditions when investors trade repo $f$ are :

$$\begin{aligned}
-p_1 + E[p_2(s)v'(c_2^1(s)] + \gamma_1^1 &= 0, \\
-p_F + E\left[f(s)v'(c_2^1(s)]\right] + \gamma_1^1 &= 0. \\
-p_1 + E\left[p_2(s)u'(c_2^2(s))\right] + \gamma_1^2 &= 0, \\
-p_F + E\left[f(s)u'(c_2^2(s))\right] &= 0. \\
-p_2(s)v'(c_2^1(s)) - s &= 0, \\
\xi_1^1 \xi_1^2 &= 0, \\
c_2^1(s) &= \omega - f(s)b^{12}, \\
c_2^2(s) &= \omega + f(s)b^{12}
\end{aligned}$$

The fourth equality derives from the fact that investor 1 will be the marginal holder of the asset into period 3. We can derive the marginal willingness to pay for any contract $\tilde{f} \in \mathcal{F}_{12}(\mathbf{p}_2)$ for both investors. In other words, we derive the minimum (resp. maximum) price $\tilde{q}^1_{12}(\tilde{f})$ and $\tilde{q}^2_{12}(\tilde{f})$ at which investor 1 (resp. investor 2) is ready to sell (resp. to buy) an infinitesimal amount of contract $\tilde{f}$.

$$\tilde{q}^1_{12}(\tilde{f}) = E\left[\tilde{f}(s)v'(c^1_2(s))\right] + \gamma^1_1$$
$$\tilde{q}^2_{12}(\tilde{f}) = E\left[\tilde{f}(s)u'(c^2_2(s))\right]$$

investor do not trade contract $\tilde{f}$ in equilibrium if and only if:

$$\tilde{q}^2_{12}(\tilde{f}) \leq \tilde{q}^1_{12}(\tilde{f}) \tag{30}$$

Indeed, if this inequality holds, there is an equilibrium price $\tilde{q}_{12}(\tilde{f}) \in [\tilde{q}^1_{12}(\tilde{f}), \tilde{q}^2_{12}(\tilde{f})]$ such that investors' optimal trade in $\tilde{f}$ is 0. We will use this inequality to show that the equilibrium $f$ is the contract characterized in Proposition 1.

### B.1.1 Characterization of the equilibrium repo contract

There are two cases.

i) $\gamma^1_1 = 0$ : investor 1 is unconstrained.

Then investors 1 and 2's (marginal) valuation for any contract $\tilde{f} \in \mathcal{F}_1(\mathbf{p}_2)$ must coincide, that is:

$$E\left[\tilde{f}(s)u'(c^2_2(s))\right] = E\left[\tilde{f}(s)v'(c^1_2(s))\right] \tag{31}$$

where $c^2_2(s) = \omega + f(s)b^{12}$. Suppose there is an open interval $(s_1, s_2) \in \mathcal{S}$ such that for all $s \in (s_1, s_2)$, $u'(c^2_2(s)) - v'(c^1_2(s)) \neq 0$ and has a constant sign. Let us then consider the piece-wise linear schedule $\tilde{f}$ such that $\tilde{f}(\underline{s}) = \tilde{f}(s_1) = \tilde{f}(s_2) = \tilde{f}(\bar{s}) = 0$ and $\tilde{f}(s_1/2{+}s_2/2) = s_1$. The schedule $\tilde{f} \in \mathcal{F}_1$ would violate equality (31). It means that there cannot be an open interval on which $u'(c^2_2(s)) - v'(c^1_2(s)) \neq 0$. Hence, by continuity, we must have for all $s \in \mathcal{S}$, $u'(c^2_2(s)) = v'(c^1_2(s))$, that is $c^2_2(s) = c^2_{2,*}$. This means that $f$ is constant and in particular that investor 2 can finance $c^2_{2,*}$ in the lowest state $\underline{s}$ so that $s^* \leq \underline{s}$. In that case, although the equilibrium allocation is unique, the contracts traded are not. The expression of $c_2^2(s)$ only pins down[27] the product $b^{12}f$, and the repurchase price $f$ may lie anywhere in the interval $[\frac{s^*}{(1-\theta_1)v'(c^1_{2,*})}, \frac{\underline{s}}{(1-\theta_1)v'(c^1_{2,*})}]$ .

ii) $\gamma_1^1 > 0$: investor 1 is constrained.

This means that $b^{12} = a$. Rewriting (30) using equilibrium conditions, we obtain:

$$E\left[\left(f(s) - \tilde{f}(s)\right)\left(u'(c_2^2(s)) - v'(c_2^1(s))\right)\right] \geq 0 \tag{32}$$

Let us now define a partition of $\mathcal{S}$ as follows

$$\mathcal{S}^+(\mathbf{p}_2) = \left\{ s \in \mathcal{S} \mid \omega + a\frac{p_2(s)}{1-\theta_1} \geq c^2_{2,*} \right\} \quad \mathcal{S}^-(\boldsymbol{p}_2) = \mathcal{S} \backslash \mathcal{S}^+(\mathbf{p}_2)$$

Hence, $\mathcal{S}^+(\mathbf{p}_2)$ is the union of intervals (by continuity) where the first best allocation is attainable given $\mathbf{p}_2$ . We have $\mathcal{S}^+(\mathbf{p}_2) \cup \mathcal{S}^+(\mathbf{p}_2) = \mathcal{S}$ also by continuity. We argue first that $f(s) = s^*/[(1-\theta_1)v'(c^1_{2,*})]$ for $s \in \mathcal{S}^+(\mathbf{p}_2)$ . If $f$ lies below this constant, by definition of $s^*$ , we have $u'(c_2^2(.)) - v'(c_2^1(.)) > 0$ . Any $\tilde{f}$ lying slightly above $f$ would then violate (32). A similar argument can be applied to show that $f$ cannot lie above $p_2(s^*)/(1-\theta_1)$ for $s \in \mathcal{S}^+(\mathbf{p}_2)$ . Now, we argue that $f(s) = p_2(s)/(1-\theta_1)$ for $s \in \mathcal{S}^-(\mathbf{p}_2)$ . If not, for all $s \in \mathcal{S}^-(\mathbf{p}_2)$, $u'(c_2^2(.)) - v'(c_2^1(.)) > 0$ so that any feasible schedule $\tilde{f}$ above $f$ would again violate (32). Hence, we have fully defined the equilibrium $f$ as a function of $\mathbf{p}_2$.

### B.1.2 Characterization of the spot market price

We now characterize the fixed point defining equilibrium $\mathbf{p}_2$. Given equilibrium trades and the equilibrium contract traded, we have:

$$\begin{cases} p_2(s)v'\left(\omega - a\frac{p_2(s)}{1-\theta_1}\right) = s & s \in \mathcal{S}^-(\mathbf{p}_2) \\ p_2(s)v'(c^1_{2,*}) = s & s \in \mathcal{S}^+(\mathbf{p}_2) \end{cases}$$

[27] In addition, investor 2 could also buy the asset spot to sell it in a repo $F_2$. In any case, having investor 1 sell $a$ units of contract $\bar{p} = s^*/(1-\theta)$ is an equilibrium since investors do not (strictly) want to trade another contract.

We have that $c_2^1(s) < c_{2,*}^1$ for $s \in \mathcal{S}^-(\mathbf{p}_2)$. Suppose there exists $s^+ \in \mathcal{S}^+(p_2)$. Since $p_2(s) > p_2(s^+)$ for $s > s^+$, we have that $[s^+, \bar{s}] \in \mathcal{S}^+(\mathbf{p_2})$. In this case, $\mathcal{S}^+(\mathbf{p}_2)$ is an interval containing the larger elements of $\mathcal{S}$. We are left to show that its minimal element is $s^*$ defined in (14). Clearly, $s^* \in \mathcal{S}^+(\mathbf{p_2})$. Consider now $\hat{s} < s^*$. By definition of $s^*$, we have that

$$\omega + a\frac{\hat{s}}{v'(c_{2,*}^1)(1-\theta_1)} < c_{2,*}^2$$

In words, the first best allocation cannot be reached if the spot market price is equal to its "fundamental value" that is $p_2(\hat{s}) = \hat{s}/v'(c_{2,*}^1)$. This means that $\hat{s} \in \mathcal{S}^-(\mathbf{p}_2)$ as otherwise, we would have $f(\hat{s}) = s^*/(1-\theta_1)$ and $p_2(\hat{s}) = \hat{s}/v'(c_{2,*}^1)$.

To conclude, the equilibrium contract $f$ ands spot market price $\mathbf{p}_2$ verify the following equations

$$\text{If} \quad s < s^*, \quad \begin{cases} p_2(s)v'\left(\omega - a\frac{p_2(s)}{1-\theta_1}\right) - s & = 0 \\ f(s) & = \frac{p_2(s)}{1-\theta_1} \end{cases}$$

$$\text{If} \quad s \geq s^*, \quad \begin{cases} p_2(s) & = s/v'(c_{2,*}^1) \\ f(s) & = \frac{p_2(s^*)}{1-\theta_1} \end{cases}$$

### B.1.3 No default-prone contracts

Consider now a repo contract $\tilde{f}$ such that investor 1 defaults in some states of the world, that is $\tilde{f}(s)$ violates (3) for some $s$. If $f$ is the equilibrium contract, we can focus on contracts such that $\tilde{f}(s) = f(s)$ for $s \geq s^*$ since it is not possible to improve over $f$ on this region. Let us now define $\mathcal{S}_d = \left\{ s \in [\underline{s}, s^*] \mid \tilde{f}(s) \text{ violates (3)} \right\}$ and $\mathcal{S}_{nd} = [\underline{s}, s^*] \backslash \mathcal{S}_d$. investors do not trade contract $\tilde{f}$ in equilibrium if and only if

$$\int_{\mathcal{S}_{nd}} \left(f(s) - \tilde{f}(s)\right)\left(u'(c_2^2(s)) - v'(c_2^1(s))\right) \mathrm{d}G(s) +$$
$$\int_{\mathcal{S}_d} \left(f(s) - \alpha\tilde{f}(s) - (1-\alpha)p_2(s)\right)\left(u'(c_2^2(s)) - v'(c_2^1(s))\right) \mathrm{d}G(s) + \pi\int_{\mathcal{S}_d} \tilde{f}(s)v'(c_2^1(s))\mathrm{d}G(s) \geq 0$$

The first line is condition (32) for a no-default contract. The second line corresponds to the states $\mathcal{S}_d$ where the borrower defaults. Observe that the realized payoff to the lender is only $p_2(s)+\alpha(\tilde{f}(s)-p_2(s))$. In addition, the borrower incurs the non-pecuniary cost (the term proportional to $\pi$). This inequality holds if

$$\int_{\mathcal{S}_d} (f(s)-(1-\alpha)p_2(s))\left(u'(c_2^2(s))-v'(c_2^1(s))\right)\mathrm{d}G(s) \geq \int_{\mathcal{S}_d} \alpha\tilde{f}(s)\left(u'(c_2^2(s))-v'(c_2^1(s))\right)\mathrm{d}G(s) - \pi\int_{\mathcal{S}_d} \tilde{f}(s)v'(c_2^1(s))\mathrm{d}G(s)$$

Using that $u'(c_2^2(s)) \leq u'(\omega)$ and $v'(c_2^1(s)) \geq v'(\omega)$ for $s \in \mathcal{S}_d \subset [\underline{s}, s^*]$, we derive the following upper bound for the right hand side:

$$[\alpha(u'(\omega)-v'(\omega))-\pi v'(\omega)]\int_{\mathcal{S}_d} \tilde{f}(s)\mathrm{d}G(s)$$

under assumption (5), this term is negative. Also, since $f(s)=\frac{p_2(s)}{1-\theta_i} > (1-\alpha)p_2(s)$ in the region $\mathcal{S}_d$, the left-hand side is positive. Hence we proved that the inequality above holds and that investors do not want to trade default-prone repo contracts.

## B.2 Proof of Proposition 3

*Proof.* Building on the case with one asset, we can characterize the equilibrium as follows. Define $s^{**}$ as the minimal state where the first best allocation can be reached.

$$\omega + \frac{a\rho_A(s^{**})+b\rho_B(s^{**})}{(1-\theta_1)v'(c_{2,*}^1)} = c_{2,*}^2.$$

Then the repayment schedule for asset $i$ is

$$f_i(s) = \begin{cases} \dfrac{p_{2,i}(s)}{1-\theta} & \text{for } s \leq s^{**}, \\ \dfrac{\rho_i(s^{**})}{(1-\theta)v'(c_{2,*}^1)} & \text{for } s \geq s^{**}. \end{cases}$$

where $(p_{2,A}(s), p_{2,B}(s))$ are the spot market prices of asset $A$ and $B$ respectively in period 2, state $s$. They are defined as follows for $i = A, B$:

$$\begin{cases} p_{2,i}(s)v'\left(\omega + \frac{ap_{2,A}(s)+bp_{2,B}(s)}{(1-\theta)}\right) - \rho_i(s) = 0 & s \leq s^{**} \\ p_{2,i}(s)v'(c^1_{2,*}) = \rho_i(s) & s > s^{**} \end{cases}$$

The liquidity premium for asset $i = A, B$ is

$$\mathcal{L}_i = \int_{\underline{s}}^{s^{**}} \frac{\rho_i(s)}{1-\theta}\left[\frac{u'\left(c^2_2(s)\right)}{v'(c^1_2(s))} - 1\right] dF(s)$$

Hence,

$$\begin{aligned} \cdot\mathcal{L}_{A,B} &= \mathcal{L}_A - \mathcal{L}_B \\ &= \int_{\underline{s}}^{s^{**}} \frac{s-\rho_\alpha(s)}{1-\theta}\left[\frac{u'\left(c^2_2(s)\right)}{v'(c^1_2(s))} - 1\right] dF(s) \\ &= -\frac{\alpha}{1-\theta}\int_{\underline{s}}^{s^{**}} (s-\mathbb{E}[s])\left[\frac{u'\left(c^2_2(s)\right)}{v'(c^1_2(s))} - 1\right] dF(s) \\ &> 0 \end{aligned}$$

where the inequality follows from the fact that the integral is negative over the integration range.

The haircut as a function of $\alpha$ is:

$$\begin{aligned} \mathcal{H}_i(\alpha) &= p_{1,i} - q_i \\ &= \mathbb{E}\left[(p_{2,i}(s) - f_i(s))v'(c^1_2(s))\right] \\ &= \mathbb{E}[p_{2,i}(s)v'(c^1_2(s))] - \int_{\underline{s}}^{s^*} \frac{p_{2,i}(s)}{1-\theta_1}v'(c^1_{2,s})\mathrm{d}F(s) - \int_{\underline{s}}^{s^*} \frac{p_{2,i}(s^*)}{1-\theta_1}v'(c^1_{2,*})\mathrm{d}F(s) \\ &= \mathbb{E}[s] - \int_{\underline{s}}^{s^{**}} \frac{\rho_i(s)}{1-\theta}dF(s) - \int_{s^{**}}^{\bar{s}} \frac{\rho_i(s^{**})}{1-\theta}dF(s) \\ &= \mathbb{E}[s] - \int_{\underline{s}}^{s^{**}} \frac{(1+\alpha_i)s - \alpha_i\mathbb{E}[s]}{1-\theta}dF(s) - \int_{s^{**}}^{\bar{s}} \frac{(1+\alpha_i)s^{**} - \alpha_i\mathbb{E}[s]}{1-\theta}dF(s) \\ &= \mathbb{E}[s] + \frac{\alpha_i\mathbb{E}[s]}{1-\theta} - \frac{(1+\alpha_i)}{1-\theta}\left[\int_{\underline{s}}^{s^{**}} s dF(s) + \int_{s^{**}}^{\bar{s}} s^{**}dF(s)\right] \end{aligned}$$

The term in brackets is less than $\mathbb{E}[s]$ therefore, for all assets $A$ and $B$ such that $\alpha_A < \alpha_B$ we obtain

$$\mathcal{H}_A < \mathcal{H}_B$$

i.e. the safe asset always commands a lower haircut than the risky asset. □

## B.3 Proof of Proposition 4

*Proof.* Given that $\nu_1 = 0$, thee same arguments apply to establish that investor 2 does not borrow in a repo so that we need to consider only one repo contract $f(\nu_2) \in \mathcal{F}_{12}(\mathbf{p}_2)$. However, spot trades may be different from zero because investor 2 can now re-sell collateral pledged by investor 1. We guess and verify that investors may not reach the first-best allocation. This implies that collateral constraints bind:

$$a_1^1 = b^{12} \tag{33}$$

$$a_1^2 = -\nu_2 \ell^{21} \tag{34}$$

Using clearing in the spot market, we have $a_1^1 + a_1^2 = a$. Market clearing for for repo requires $b^{12} = \ell^{21}$. Summing (33) and (34) we obtain

$$a_1^1 = b^{12} = \frac{a}{1-\nu_2}$$
$$a_1^2 = -\nu b^{12} = -\frac{\nu_2}{1-\nu_2} a$$

We can thus write investor 2 consumption as

$$c_2^2(s) = \omega + \frac{a}{1-\nu_2}(f(s,\nu_2) - \nu_2 p_2(s))$$

We can then adapt the proof of the no re-use case. As before, $f(s,\nu_2)$ must be such that $c_2^2(s) = c_{2,*}^2$ whenever possible and equal to the no default limit $p_2(s)/(1-\theta_1)$ otherwise. Let us define $s^*(\nu_2)$ as the minimal state where the first-best level of consumption can be attained. Plugging $f(s,\nu_2) = p_2(s)/(1-\theta_1)$ in the equality above and using $p_2(s) = s/v'(c_2^1(s))$, we obtain

$$\omega + \frac{as^*(\nu_2)}{(1-\nu_2)v'(c_{2,*}^1)}\left[\frac{1}{1-\theta_1} - \nu_2\right] = c_{2,*}^2.$$

We thus have:

$$f(s,\nu_2) = \begin{cases} \frac{p_2(s)}{1-\theta_1} & \text{if } s < s^*(\nu_2) \\ \frac{s^*(\nu_2)}{(1-\theta_1)v'(c_{2,*}^1)} + \frac{\nu(s-s^*(\nu_2))}{v'(c_{2,*}^1)} & \text{if } s \geq s^*(\nu_2) \end{cases}$$

Since $v \to \frac{1-(1-\theta)v}{1-v}$ is increasing in $v$, $s^*(\nu_2)$ is decreasing in $v_2$ and $\lim_{\nu_2 \to 1} s^*(\nu_2) < 0$. Hence there exists $\nu^* \in (0,1)$ such that $s^*(\nu^*) = \underline{s}$. To find the expression for $\nu^*$ notice that

$$c_{2,*}^2 = \omega + M_{12}(\nu_2)p_2(s^*(\nu_2)) = \omega + M_{12}(0)p_2(s^*(0))$$

and using $s^*(0) = s^*$ as well as $s^*(v^*) = \underline{s}$ we find $\nu^* = \frac{s^* - \underline{s}}{s^* - (1-\theta_1)\underline{s}}$.

We now verify that the lender does not default given $f(s,\nu_2)$. Recall that the no default constraint is

$$f(s,\nu_2) \geq \frac{v_2}{1+\theta_2}p_2(s)$$

which is clearly satisfied by the candidate $f(s,\nu_2)$.

*Proof of the Remark*

Assuming now that $\nu_1 > 0$, we provide a formal argument for the claim in the Remark below Proposition 3. investor 2 does not want to sell in a repo if for all $\tilde{f}_{21} \in \mathcal{F}_{21}(\mathbf{p}_2)$, we have:

$$E[\tilde{f}_{21}(s)u'(c_2^2(s))] + \gamma_1^2 \geq E[\tilde{f}_{21}(s)v'(c_2^1(s))] + \nu_1\gamma_1^1$$

Using the equilibrium characterization, we obtain the following inequality:

$$E\left[\tilde{f}_{21}(s)\left(u'(c_2^2(s)) - v'(c_2^1(s))\right)\right] \geq \frac{1}{1-\nu_2}E\left[(\nu_1(f(s,\nu_2) - \nu_2 p_2(s)) - f(s,\nu_2) + p_2(s))\left(u'(c_2^2(s)) - v'(c_2^1(s))\right)\right]$$

Using the expression for $f(s,\nu_2)$ we derived and $\tilde{f}_{21} = \nu_1 p_2(s)/(1+\theta_1)$ (the contract for which the inequality above is the most difficult to satisfy), we obtain:

$$\begin{aligned} \frac{\nu_1}{1+\theta_1} &\geq \frac{(1-\nu_1\nu_2)(1-\theta_1)-(1-\nu_1)}{(1-\theta_1)(1-\nu_2)} \\ \Leftrightarrow \quad \nu_1(1-\nu_2)(1-\theta_1) &\geq \nu_1(1-\nu_2)(1+\theta_1)-\theta_1(1-\nu_1\nu_2)(1+\theta_1) \\ \Leftrightarrow \quad \theta_1(1-\nu_2\nu_1)(1+\theta_1) &\geq 2\nu_1(1-\nu_2)\theta_1 \end{aligned}$$

The opposite of this inequality gives condition (21). □

## B.4 Proof of Proposition 5 and 6

### B.4.1 Equilibrium trades with 3 investors.

We prove Proposition 5 and 6 as corollaries of the following proposition 8. We first define $s^*_{B2}(b)$ implicitly as:

$$u'\left(\omega + \frac{bs^*}{(1-\nu_2)\delta}\left[\frac{1}{1-\theta_B} - \nu_2\right]\right) = \delta_B,$$

For a given amount of asset $b$ used for the transaction, $s^*_{B2}(b)$ is the threshold in $s$ above which marginal rates of substitution between investors $B$ and 2 can be equalized. Define also the repo contract $f_{B2}(b)$ implicitly as a function of the amount borrowed $b$:

$$f_{\text{B2}}(b,\nu_2,s) = \begin{cases} \frac{p_2(s)}{1-\theta_B} & \text{if } s < s^*_{B2}(b) \\ \frac{s^*(b,\theta_i,\nu)}{\delta(1-\theta_B)} + \frac{\nu_2(s-s^*(b,\theta_i,\nu_2))}{\delta} & \text{if } s \geq s^*_{B2}(b) \end{cases} \tag{35}$$

We define the repo contract that investors 1 and $B$ will trade in equilibrium if any:

$$f_{1B}(s) = \frac{p_2(s)}{1-\theta_1} \qquad \forall s \in [\underline{s}, \bar{s}] \tag{36}$$

Finally observe that $p_2(s) = s/\delta$ since investor 1 holds the asset into period 3.

**Proposition 8.** *Let us define $\hat{b}$ implicitly as:*

$$\int_{\underline{s}}^{s^*_{B2}(\hat{b})} \left[ u'\left( \omega + \frac{\hat{b}}{1-\nu_2}\left[\frac{1}{1-\theta_B} - \nu_2\right] p_2(s) \right) - \delta_B \right] \mathrm{p_2(s)d}F(s) = \frac{(\delta_B - \delta)\theta_1}{(1-\nu_B)(1-\theta_1)} \frac{(1-\theta_B)(1-\nu_2)}{1-(1-\theta_B)\nu_2} E[p_2(s)] \tag{37}$$

*Four cases are then possible:*

*1) $\hat{b} > a$. investor 1 sells the asset spot to $B$ who borrows $b_*^{B2} = a/(1-\nu_2)$ from investor 2 using repo $f_{B2}(a, \nu_2)$.*

*2) $\hat{b} \in [\nu_H a, a]$. investor 1 uses a combination of a spot and repo sale with $f_{1B}$ with $B$. investor $B$ borrows $b^{B2} = \hat{b}/(1-\nu_2)$ from investor 2 using repo $f_{B2}(b^{B2}, \nu_2)$.*

*3) $\hat{b} \in [0, \nu_H a]$. investor 1 borrows from $B$ using repo $f_{1B}$ and $B$ borrows $b^{B2} = \hat{b}/(1-\nu_2)$ from investor 2 using repo $f_{B2}(b^{B2}, \nu_2)$.*

*4) $\hat{b} < 0$. investor 1 sells the asset using repo $f_{1B}$ to investor $B$. investor 2 does not trade, that is $b^{B2} = 0$.*

*In cases 2 and 3, the following condition is necessary :*

$$\frac{1}{1-\theta_B} - \frac{1}{1-\theta_1} \geq (1-\nu_B)\frac{1-\nu_2(1-\theta_B)}{(1-\nu_2)(1-\theta_B)}$$

*In all cases the amount $b_*^{1B}$ borrowed by investor 1 from $B$ is given by:*

$$b_*^{1B} = \frac{a - (1-\nu_2) b_*^{B2}}{1-\nu_B} \tag{38}$$

*Proof.* Under our conjecture, investors 1 and $B$ may trade in a repo $f_{1B}$ and investors $B$ and 2 can trade in a repo $f_{B2}$. All investors may trade in the spot market.

**Step 1: investors problem and first order conditions**

investor 1 chooses spot trade $a_1^1$ and repo trade $b^{1B}$ with investor $B$ to solve:

$$\max_{a_1^1, b^{1B}} \omega + p_1(a - a_1^1) + q_{1B}b^{1B} + E\left[\delta\left(\omega + p_2(s)a_1^1 - f_{1B}(s)b^{1B}\right) + a.s\right]$$

$$\begin{aligned} \text{s.to} \qquad a_1^1 \geq &\ b^{1B} \quad (\gamma_1^1) \\ b^{1B} \geq &\ 0 \quad (\xi_{1B}) \end{aligned}$$

where we used the fact that investor 1 will hold the asset into period 3, that is $a_2^1(s) = a$. investor $B$ chooses spot trade $a_1^B$, repo lending $l^{1B}$ to investor $B$ and repo borrowing $b^{B2}$ from investor 2 to solve:

$$\begin{aligned} &\max_{a_1^B, l^{B1}, b^{B1}} \omega - p_1 a_1^B - q_{1B}\ell^{B1} + q_{B2}b^{B2} \\ &+\delta_B E\left[\omega + p_2(s)a_1^B + f_{1B}(s)\ell^{B1} - f_{B2}(s)b^{B2}\right] \\ &\text{s.to} \qquad a_1^B + \nu_B \ell^{B1} \geq \ b^{B2} \quad (\gamma_1^B) \\ &\qquad\qquad \ell^{B1} \geq \ 0 \quad (\xi_{B1}) \\ &\qquad\qquad b^{B2} \geq \ 0 \quad (\xi_{B2}) \end{aligned}$$

investor 2 chooses spot trade $a_1^2$ and repo lending $l^{B2}$ to investor $B$ to solve:

$$\max_{a_1^2, l^{2B}} \omega - p_1 a_1^2 - q_{B2}\ell^{2B} + E\left[u\left(\omega + s a_1^2 + f_{B2}(s)\ell^{2B}\right)\right]$$

$$\begin{aligned} \text{s.to} \qquad a_1^2 \ +\nu_2\ell^{2B} &\geq 0 \quad (\gamma_1^2) \\ \ell^{2B} \qquad &\geq 0 \quad (\xi_{2B}) \end{aligned}$$

Let us now write down the first order conditions for our 3 investors:

$$\begin{aligned}
-p_1 + \delta E[p_2(s)] + \gamma_1^1 &= 0 && (39)\\
q_{1B} - \delta E[f_{1B} s)] - \gamma_1^1 + \xi_{1B} &= 0 && (40)\\
-p_1 + \delta_B E[p_2(s)] + \gamma_1^B &= 0 && (41)\\
-q_{1B} + \delta_B E[f_{1B}(s)] + \nu_B \gamma_1^B + \xi_{B1} &= 0 && (42)\\
+q_{B2} - \delta_B E\left[f_{B2}(s)\right] - \gamma_1^B &= 0 && (43)\\
-p_1 + E\left[p_2(s) u'(c_2^2(s))\right] + \gamma_1^2 &= 0 && (44)\\
-q_{B2} + E\left[f_{B2}(s) u'(c_2^2(s))\right] + \nu_2 \gamma_1^2 &= 0 && (45)
\end{aligned}$$

Market clearing implies that $b^{ij} = \ell^{ji}$ for each pair of investors $(i, j)$. Hence, we only use the notation $b$ in the following. Observe that we included the Lagrange multipliers $\xi_{1B}$ and $\xi_{B1}$ for the positivity constraint on the repo trade between 1 and $B$. Quick manipulations of equations (39) to (45) give the following expressions for the Lagrange multipliers associated to the collateral constraints:

$$\gamma_1^2 = \frac{1}{(1-\nu_2)} E\left[\left(f_{B2}(s) - p_2(s)\right)\left(u'(c_2^2(s)) - \delta_B\right)\right] \quad (46)$$

$$\gamma_1^B = \frac{1}{(1-\nu_2)} E\left[\left(f_{B2}(s) - \nu_2 p_2(s)\right)\left(u'(c_2^2(s)) - \delta_B\right)\right] \quad (47)$$

$$\gamma_1^1 = \gamma_1^B + (\delta_B - \delta) E[p_2(s)] \quad (48)$$

so that $\gamma_1^1 \geq \gamma_1^B \geq \gamma_1^2$. We guess and verify that $\gamma_1^2 > 0$. This implies that all collateral constraints bind:

$$\begin{aligned}
a_1^1 &= b^{1B}\\
a_1^B + \nu_B b^{1B} &= b^{B2}\\
a_1^2 + \nu_2 b^{B2} &= 0
\end{aligned}$$

while market clearing for the asset yields:

$$a_1^1 + a_1^B + a_1^2 = a$$

Using this last equation together with the collateral constraints above, we obtain equation (38), that is

$$a = (1 - \nu_B) b^{1B} + (1 - \nu_2) b^{B2}$$

Finally, we can write the consumption of investor 2 in period 2:

$$c_2^2(s) = \omega + b^{B2}(f_{B2}(s, b^*, \nu_2) - \nu_2 p_2(s)) \tag{49}$$

where $b^* := (1 - \nu_2) b^*_{B2}$ is the physical amount of asset used between $B$ and 2.

**Step 2 : Determination of equilibrium trades and $b^*$.**

We now characterize the repo contracts (if any) traded by investors 1 and $B$ and $B$ and 2. We will also pin down $b^*$, the amount of asset used to support the trade between $B$ and 2.

*Equilibrium repo contract $f_{B2}$ between B and 2.*

For a given value of $b^*$, investors $B$ and 2 trade as in the previous section replacing $a$ by $b^*$. Using our previous results, the equilibrium repo contract is $f_{B2}(b^*, \nu_2)$ defined in (35).

*Equilibrium trade between* 1 *and* $B$

We examine the cases where investors only trade spot and when they use a repo in turn.

i) $b_{1B} = 0$, that is investors 1 and $B$ trade only spot, when no repo contract $\tilde{f}_{1B}$ is profitable:

$$\delta E[\tilde{f}_{1B}(s)] + \gamma_1^1 \geq \delta_B E[\tilde{f}_{1B}(s)] + \nu_B \gamma_1^B$$

The constraint is tighter with $\tilde{f}_{1B}(s) = p_2(s)/(1 - \theta_1)$ and we obtain:

$$\frac{(\delta_B - \delta)\,\theta_1}{1 - \theta_1} E[p_2(s)] \leq (1 - \nu_B)\gamma_B^1 \tag{50}$$

From the collateral constraints, we obtain $a_1^1 = 0$ and $a_1^B = \frac{a}{1-\nu_2}$. This implies that $b^* = a$ and $b^{B2} = \frac{a}{1-\nu_2}$. investor 2 consumption is

$$c_2^2(s) = \begin{cases} \omega + \frac{ap_2(s)}{1-\nu_2}\left[\frac{1}{1-\theta_B} - \nu_2\right] & \text{if } s < s^*_{B2}(a) \\ c^2_{2,*} & \text{if } s \geq s^*_{B2}(a) \end{cases}$$

Using the equation for $\gamma_1^B$ in (47), expression (50) becomes

$$\frac{1-(1-\theta_B)\nu_2}{(1-\theta_B)(1-\nu_2)}\int_{\underline{s}}^{s^*_{B2}(a)} \left[u'\left(c_2^2(s)\right) - \delta_B\right] p_2(\mathrm{s})\mathrm{d}G(s) \geq \frac{(\delta_B - \delta)\theta_1}{(1-\nu_B)(1-\theta_1)}E[p_2(s)]$$

The mapping

$$x \to \int_{\underline{s}}^{s^*_{B2}(x)} \left[u'\left(c_2^2(s)\right) - \delta_B\right] p_2(\mathrm{s})\mathrm{d}G(s)$$

is decreasing. Hence, the inequality above is equivalent to $\hat{b} > a$ (case 1).

ii) $b_{1B} > 0$, that is investors trade repo. We can characterize $f_{1B}$ as before, using that investors 1 and $B$ should not be willing to trade another repo contract $\tilde{f}_{1B}$. We obtain:

$$f_{1B}(s) = \frac{p_2(s)}{1-\theta_L}, \quad \forall s.$$

This implies, using (39)-(42) that:

$$\gamma_1^B = \frac{\theta_1(\delta_B - \delta)}{(1-\theta_1)(1-\nu_B)}E[p_2(s)]$$

Using the expression for $\gamma_1^B$ in (47), we obtain the following equality:

$$\frac{1-(1-\theta_B)\nu_2}{(1-\theta_B)(1-\nu_2)}\int_{\underline{s}}^{s^*_{B2}(b^*)} \left[u'\left(c_2^2(s)\right) - \delta_B\right] p_2(\mathrm{s})\mathrm{d}G(s) = \frac{(\delta_B - \delta)\theta_1}{(1-\nu_B)(1-\theta_1)}E[p_2(s)]$$

which, together with (49), pins down the amount $b^*$. Our conjectured pattern of trades can be an equilibrium if and only if the solution is feasible that is $b^* \in [0, a]$. Suppose first that $b^* \in [0, \nu_B a]$. Then from equation (38), we have $b_{1B} > a$ so that $a_1^1 > a = a_0^1$ using investor 1 collateral constraint. Hence, investor 1 buys the asset spot and only sells it to $B$ using repo $f_{1B}$ (case 3). If $b^* \in [\nu_B a, a]$, we have $a_1^1 < a$, so that investor 1 also sells the asset spot to investor $B$ (case 2).

**Step 3 : No profitable contract between investors 1 and 2**

We need to check that intermediation is optimal, that is investors 1 and 2 do not want to trade any repo contract $\tilde{f}_{12}$. This requires

$$\delta E[\tilde{f}_{12}(s)] + \gamma_1^1 \geq E[\tilde{f}_{12}(s) u'(c_2^2(s))] + \nu_2 \gamma_1^2$$

We can rewrite the condition as

$$(1-\nu_2)\gamma_1^2 \geq E\left[\left(\tilde{f}_{12}(s) - p_2(s)\right)\left(u'(c_2^2(s)) - \delta\right)\right]$$

Using the expression of $\gamma_1^2$, we obtain:

$$E\left[(f_{B2}(s) - p_2(s))\left(u'(c_2^2(s)) - \delta_B\right)\right] \geq E\left[\left(\tilde{f}_{12}(s) - p_2(s)\right)\left(u'(c_2^2(s)) - \delta\right)\right]$$

or

$$E\left[\left(f_{B2}(s) - \tilde{f}_{12}(s)\right)\left(u'(c_2^2(s)) - \delta_B\right)\right] \geq \frac{\theta_1(\delta_B - \delta)}{1-\theta_1} E[p_2(s)]$$

We plug $\tilde{f}_{12} = p_2(s)/(1-\theta_1)$ in the left hand side to find the tightest bound:

$$\left(\frac{1}{1-\theta_B} - \frac{1}{1-\theta_1}\right) \int_{\underline{s}}^{s^*(b^*,,\theta,\nu_2)} p_2(s)\left(u'\left(c_2^2(s)\right) - \delta_B\right) \geq \frac{\theta_1(\delta_B - \delta)}{1-\theta_1} E[p_2(s)]$$
$$\frac{(1-\nu_2)(1-\theta_B)}{1-(1-\theta_B)\nu_2}\left[\frac{1}{1-\theta_B} - \frac{1}{1-\theta_1}\right]\gamma_1^B \geq \frac{\theta_1(\delta_B - \delta)}{1-\theta_1} E[p_2(s)]$$

where, to derive the last line, we used the expression for $\gamma_1^B$ from (47). From (39)-(42), right hand side lies below $(1-\nu_B)\gamma_1^B$ and that it is equal when $b_{1B} > 0$. In this latter case, we can rewrite the necessary condition above as

$$\left(1 - \frac{1-\theta_B}{1-\theta_1}\right)\frac{1}{1-\nu_2(1-\theta_B)} \geq \frac{1-\nu_B}{1-\nu_2}$$

This is sufficient condition (6). The condition is also necessary when $b^{1B} > 0$.

□

### B.4.2 Corollaries: Proposition 5 and 6

In Proposition 5, we assume that $\delta_B = \delta$. Then, the right hand side of (37) is zero so that the solution is $\hat{b} > a$. Proposition 5 is thus a particular instance of case 2. investor 1 sells the asset spot who re-uses it to borrow in repo $f_{B2}(a, \nu_2)$. The upper bound $a^*$ on $a$ follows from imposing $s^* > \underline{s}$ as otherwise investors would reach the first-best allocation.

Proposition *6* corresponds to case 3 where investor 1 borrows in a repo from investor $B$ who re-uses the collateral by re-selling spot and re-pledging to investor 2. As we showed, this holds if the solution to (37) is some $\hat{b} \in [0, \nu_B a]$. Using (37), we have that $\hat{b} \geq 0$ if

$$u'(\omega) - \delta_B \geq \frac{\theta_1}{(1-\nu_B)(1-\theta_1)} \frac{(1-\theta_B)(1-\nu_2)}{1-(1-\theta_B)\nu_2}(\delta_B - \delta)$$

This is equivalent to:

$$\delta_B - \delta \leq \left[1 + \frac{\theta_1}{(1-\nu_B)(1-\theta_1)} \frac{(1-\theta_B)(1-\nu_2)}{1-(1-\theta_B)\nu_2}\right]^{-1} (u'(\omega) - \delta)$$

Similarly, we have $\hat{b} \leq \nu_B a$ if

$$\int_{\underline{s}}^{s^*_{B2}(\nu_B a)} \left[u'\left(\omega + \frac{\nu_B a}{1-\nu_2}\left[\frac{1}{1-\theta_B} - \nu_2\right]\frac{s}{\delta}\right) - \delta_B\right] s \mathrm{d}F(s) \leq \frac{(\delta_B - \delta)\theta_1}{(1-\nu_B)(1-\theta_1)} \frac{(1-\theta_B)(1-\nu_2)}{1-(1-\theta_B)\nu_2}$$

so that $\delta_B - \delta$ cannot be too large either.

## B.5 Proof of the Claim in Section 6.

*Proof.* Consider a repo contract such that the repurchase price lies in the interval $(f_{min}, f_{max})$. For any such contract, there exists a threshold $s_d \in (\underline{s}, s^*)$ such that investor 1 defaults below $s_d$ and we write $f_{sd} := s_d/[\delta(1-\theta_1)]$. Let $f^1_{s_d} = \{f^1_{s_d}(s)\}_{s \in \mathcal{S}}$ be the effective cost to investor 1 while $f^2_{s_d} = \{f^2_{s_d}(s)\}_{s \in \mathcal{S}}$ is the effective payment to investor 2. We have

$$f^2_{s_d}(s) = \begin{cases} \alpha f_{s_d} + (1-\alpha)p_2(s) & \text{if} \quad s < s_d \\ f_{s_d} & \text{if} \quad s \geq s_d \end{cases}, \qquad f^1_{s_d}(s) = \begin{cases} f^1_{s_d}(s) + \pi f_{s_d} & \text{if} \quad s < s_d \\ f_{s_d} & \text{if} \quad s \geq s_d \end{cases}$$

investors 1 and 2 can trade spot and repo $f_{s_d}$. From our previous analysis, we know that an equilibrium where investor 1 sells all his asset in a repo exists if and only if $\gamma_1^2 > 0$ where $\gamma_1^2$ is the Lagrange multiplier on the short sale constraint of investor 2. Using the first order conditions with respect to spot and repo trades in $f_{s_d}$, we obtain:

$$\gamma_1^2 = E\left[\left(f_{s_d}^2(s) - p_2(s)\right) u'(c_2^2(s)) - \left(f_{s_d}^1(s) - p_2(s)\right)\delta\right]$$

where $c_2^2(s) = \omega + af_{s_d}^2(s)$. Transforming the equality above, we obtain:

$$\begin{aligned}\gamma_1^2 &= \alpha\int_{\underline{s}}^{s_d} (f_{s_d} - p_2(s))\left(u'(c_2^2(s)) - \delta\right)\mathrm{d}G(s) - \pi\delta f_{s_d} G(s_d) + \int_{s_d}^{\bar{s}} (f_{s_d} - p_2(s))\left(u'(c_2^2(s)) - \delta\right)\mathrm{d}G(s) \\ &= M(s_d)\end{aligned}$$

We are left to prove that there exists some $s_d > \underline{s}$ such that $M(s_d) > 0$. Since $M$ is continuous in $\underline{s}$, it is enough to find conditions under which $M(\underline{s}) > 0$. We have

$$\begin{aligned}M(\underline{s}) &= \int_{\underline{s}}^{\bar{s}} \left(\frac{\underline{s}}{1-\theta_1} - s\right)\left(\frac{u'(c_2^2) - \delta}{\delta}\right)\mathrm{d}G(s) \\ &= \left(\frac{u'(c_2^2) - \delta}{\delta}\right)\left(\frac{\underline{s}}{1-\theta_1} - E[s\}\right)\end{aligned}$$

where we use the fact that $c_2^2 = \omega + af_{min}$ is constant. Hence if $\underline{s} > E[s](1-\theta_1)$ we have proven the result. $\square$

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